

# Tett koblet PPP/INS for marine anvendelser

Narve S. Kjørsvik

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TerraTec's software TerraPOS has delivered state-of-the-art post-processed Global Navigation Satellite Systems (GNSS) solutions based on Precise Point Positioning (PPP) since 2006. Since 2011 the software has been extended to support integrated GNSS and inertial navigation systems (INS) processing. This allows TerraPOS to provide precise post-processed positions, velocities and attitude for kinematic platforms such as ships or aircraft.

This paper describes some implementation details, and shows initial results provided by the software using raw data collected with a commercial real-time inertial navigation system for the marine market.

*Key words:* Offshore surveying, inertial navigation, precise point positioning

*Narve S. Kjørsvik, Terratec AS, POB 513, NO-1327 Lysaker. E-Mail: narve.kjorsvik@terratec.no*

## 1 Introduction

PPP is a processing method for GNSS using undifferenced observations in conjunction with precise ephemerides and satellite clock corrections. Traditionally, differential GNSS (DGNSS) methods have been employed in applications requiring precise positioning. DGNSS relies on reference station infrastructure to compensate for common errors. PPP does not suffer from this dependence and is thus a logistically attractive alternative. The accuracy of kinematic PPP has been proved comparable to that of medium-range DGNSS (sub-dm).

Most marine surveys also require accurate attitude, usually provided by a dedicated attitude system consisting of inertial sensors (gyros and accelerometers) and dual GNSS receivers and antennas. The positioning and attitude systems are often treated as separate and then crudely integrated at a very late stage. This “loosely coupled” approach ignores possible feedback effects. This is in stark contrast to the obvious benefits of early integration, i.e., at the observation level (“tight coupling”). The excellent short-term stability of inertial sensors enables very precise prediction of the navigation states over some seconds to minutes, depending of the quality of the inertial sensors. This prediction greatly enhances error detection in GNSS data, thereby preventing contamination of the combi-

ned navigation filter. At the same time, highly accurate and long-term stable GNSS observations aid the estimation of inertial sensor biases. A fully implemented INS allows navigation output even during complete GNSS outages.

Post-processing allows more sophisticated modeling and sensor bias estimation. This is partly due to computational constraints in a low-latency real-time system, but also because some system states are only weakly observable. The observability may be enhanced by providing more accurate observations and by increasing the number of observations used to estimate a system state at a particular epoch. The accuracy of the GNSS range observations can be improved by using post-processed satellite clocks and ephemerides. When post-processing it is also possible to use a statistical estimator that allows all observations to contribute to all system states at all epochs. A post-processed solution will thus always be superior to a real-time solution. This statistical fact can be exploited in two ways: the survey requirements can be met using less expensive equipment, or the accuracy gain with existing equipment may allow new applications or more competitive survey products. Post-processing may thus also prolong the life-time of existing attitude systems, enabling them to meet future requirements. Post-processing opens for possibili-

ties to “repair” problematic survey data due to erroneously entered sensor lever-arms or mounting angles, timing biases, etc. Small errors not detectable during near real-time QC may be identifiable due to the improved accuracy of the post-processed navigation solution.

## 2 Inertial Navigation Systems

The navigation equations constitute the solution to the set of differential equations relating the sensed accelerations to the second-order time-derivative of the position. The e-frame here denotes an Earth-centered Earth-fixed frame (ECEF). The fundamental relation between the sensed specific force ( $f^s$ ), inertial acceleration ( $\ddot{x}^e$ ) and the gravitational acceleration ( $g^e$ ) is (e.g., Jekeli, 2001)

$$\ddot{x}^e = C_s^e f^s + g^e - 2 \Omega_{ie}^e \dot{x}^e \quad (1)$$

$$\frac{d}{dt} \begin{pmatrix} \delta x^e \\ \delta \dot{x}^e \\ \psi^e \end{pmatrix} = \begin{pmatrix} 0 & I & 0 \\ \Gamma^e - \Omega_{ie}^e \cdot \Omega_{ie}^e & -2 \cdot \Omega_{ie}^e & -f^e \times \\ 0 & 0 & -\Omega_{ie}^e \end{pmatrix} \cdot \begin{pmatrix} \delta x^e \\ \delta \dot{x}^e \\ \psi^e \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & -C_s^e & I \\ -C_s^e & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \delta \omega^s \\ \delta f^s \\ \delta g^e \end{pmatrix}$$

where  $\delta x^e$ ,  $\delta \dot{x}^e$  and  $\psi^e$  denote position, velocity and attitude error states respectively,  $\Gamma^e$  denotes the gravitational gradient tensor (i.e.  $\partial g^e / \partial x^e$ ),  $\delta \omega^s$  and  $\delta f^s$  denote gyro and accelerometer errors and  $\delta g^e$  denotes errors in the gravity correction.

An Extended Kalman Filter (Gelb, 1974) might be used to provide optimal estimates of the error states when fed with suitable aiding measurements such as GNSS range and range rate observations.

## 3 Precise Point Positioning

PPP is based on the principle of undifferenced GNSS observations and precise satellite orbits and satellite clock corrections (Zumberge et al., 1997 and others). A comprehensive overview is given in Kouba and Heroux (2001) in the context of the International GNSS Service (IGS, see Dow et al. (2005)).

The use of undifferenced observations in high-accuracy applications demands consi-

derable effort in modeling and estimating the host of relevant effects including satellite attitude, satellite and receiver antenna phase center variations, propagation delays and satellite and receiver hardware delays. Conformance with the conventional reference frames of the International Earth Rotation and Reference Systems Service (IERS) requires careful modeling of relativistic effects and tide and loading effects, among others (Péti and Luzum, 2010).

where  $C_s^e$  denotes the rotation matrix from the IMU (s-frame) to the e-frame,  $\Omega_{ie}^e$  denotes the earth’s angular rate in skew-symmetric matrix form and  $\dot{x}^e$  denotes velocity. In a strapdown INS the gyros are rigidly attached to the vehicle, and sense the angular velocities of the IMU relative to an inertial frame. The attitude then follows by numerical integration of the angular rates. The system of three second order differential equations in Eq. (1) is finally transformed into a system of six first order differential equations using the velocities as new variables. The specific force and gravitational acceleration constitute the forcing functions. By introducing known initial conditions, the system of differential equations may then be solved (by numerical integration) with respect to position and velocity. Perturbation of the navigation equations yields a set of linear error equations. The velocity, position and attitude error equations may then be summarized in matrix form (Jekeli, 2001) as

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## 4 The TerraPOS software

TerraPOS contains sophisticated and fully automated GNSS data editing algorithms, utilizing all available data. The current implementation allows practically all bad observations to be removed, and typically 90–95% of the cycle-slips can be correctly determined and fixed. The software adheres to the IERS and IGS conventions and implements all relevant models for state of the art PPP.

An Earth-Centered Earth-Fixed (ECEF) mechanization is used, with attitude represented using quaternions. This combined formulation is global and completely singularity-free for all positions and attitudes. Numerical integration of attitude and the navigation equations is performed using the efficient and stable Runge-Kutta algorithms.

INS processing is available in loosely coupled mode (using positions from a pre-determined GNSS solution) or in tightly coupled mode (integration at the observation level). Single or dual antenna configurations are available in tightly coupled mode. Post-processed heave is also available, but will not be further discussed here.

TerraPOS will by default employ an optimal fixed-interval smoother of the Rauch-Tung-Striebel (RTS) type (Gelb, 1974), thereby providing optimal estimation of the entire trajectory using all data.

## 5 Marine test data

The marine test data were collected by a survey vessel just outside Trondheim, Norway, as shown in Figure 1. A Kongsberg Seatex Seapath 330+ system was used to collect 1

Hz GNSS data from two antennas with fixed 2.5 m separation, and IMU raw data were sampled at 200 Hz.

The Seapath 330+ is a complete INS with real-time data output in addition to the raw data logging, specifically designed for the marine market. It features the latest IMU developed by Kongsberg Seatex, the MRU5+. The stated accuracy of the 330+ is  $0.01^\circ$  for roll/pitch and  $0.065^\circ$  for heading using 2.5 m antenna separation.

A static on-shore GNSS reference receiver sampled at 1 Hz for the purpose of generating an independent reference trajectory. The vessel was also equipped with a Sagem Sigma 40 navigation grade INS (1.5 Nautical mile/24 hours autonomous operation), This is an order of magnitude better than the Seapath, and suitable as an attitude reference.

Based on a survey over several days, a 5 hour segment of data collected November 30th has been arbitrarily chosen for analysis in this study.

## 6.1 Reference solutions

The Sigma 40 INS was aided by a single-frequency code receiver. Hence only the attitude data may serve as truth in this context. The

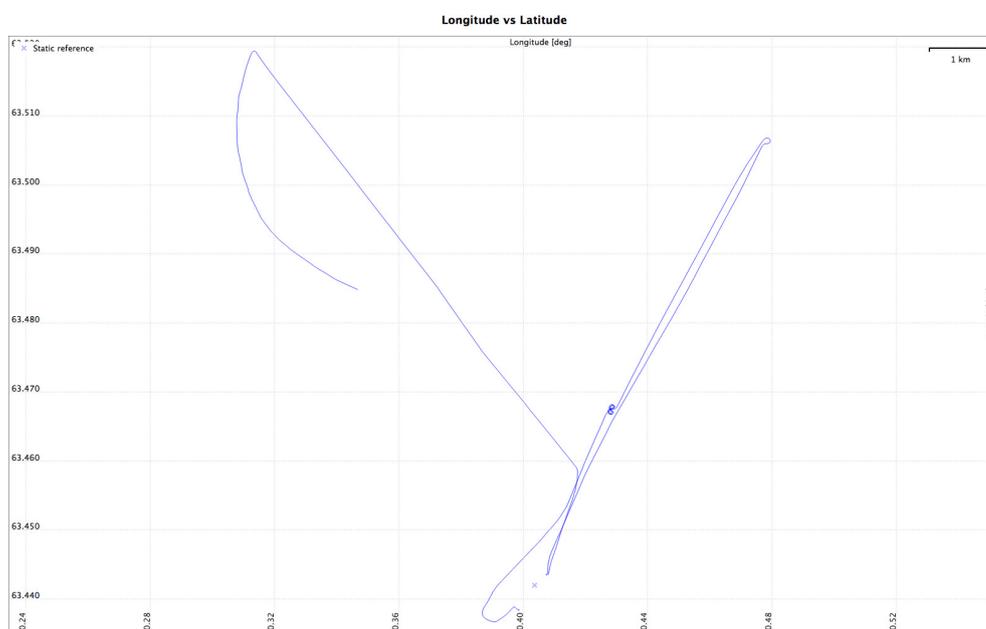


Figure 1: Overview of the test data. The cross marks the position of the reference receiver.

installation was not externally calibrated, meaning that the mean attitude differences between the Sigma 40 and the TerraPOS solution are estimated from the data and removed before comparisons are made.

Using the primary Seapath antenna and the static reference receiver, a short baseline (approximately 10 km at the most) could be estimated using a commercial differential GNSS software. The ambiguities were resolved and the stated formal precision was 15–20 mm horizontally and 30–40 mm vertically. This trajectory was finally run through TerraPOS in a loosely coupled INS processing, thus effectively attenuating the high-frequency noise in the DGNSS solution and providing approximate attitude to perform lever-arm compensation during comparisons. The estimated velocities from the combined DGNSS/INS solution are expected to be significantly better than 30 mm/s (the stated real-time accuracy of the Seapath 330+). The formal precision of the post-processed velocity solution is better than 5 mm/s.

### 6.2 Position and velocity results

A PPP solution for the secondary Seapath antenna was processed with TerraPOS with-

hout using inertial data and was compared to the reference solution. A plot of the position differences is shown in Figure 2. The RMSs of the position errors are 20 mm, 16 mm and 32 mm for north, east and height respectively. Corresponding errors at the 98% level are 46 mm, 40 mm and 86 mm respectively.

A similar comparison was also made for the PPP-estimated velocities, yielding RMS of 14 mm/s, 14 mm/s and 8 mm/s for north, east and height respectively. Corresponding errors at the 98% level are 43 mm/s, 44 mm/s and 23 mm/s respectively. A plot of the velocity errors is shown in Figure 3.

### 6.3 Attitude results

A tightly coupled PPP/INS solution was processed with TerraPOS. The attitude was compared to the reference solution and a plot of the attitude differences is shown in Figure 4. The RMSs of the attitude errors are 0.003°, 0.002° and 0.035° for roll, pitch and heading respectively. Corresponding errors at the 98% level are 0.005°, 0.008° and 0.080° respectively.

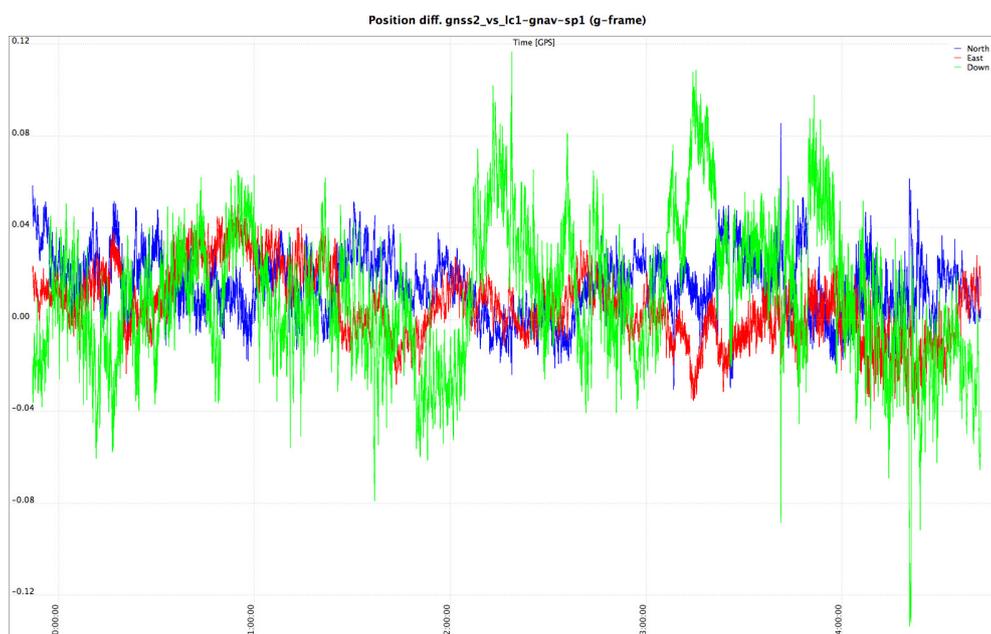


Figure 2: Position differences with respect to the ambiguity-fixed reference trajectory.

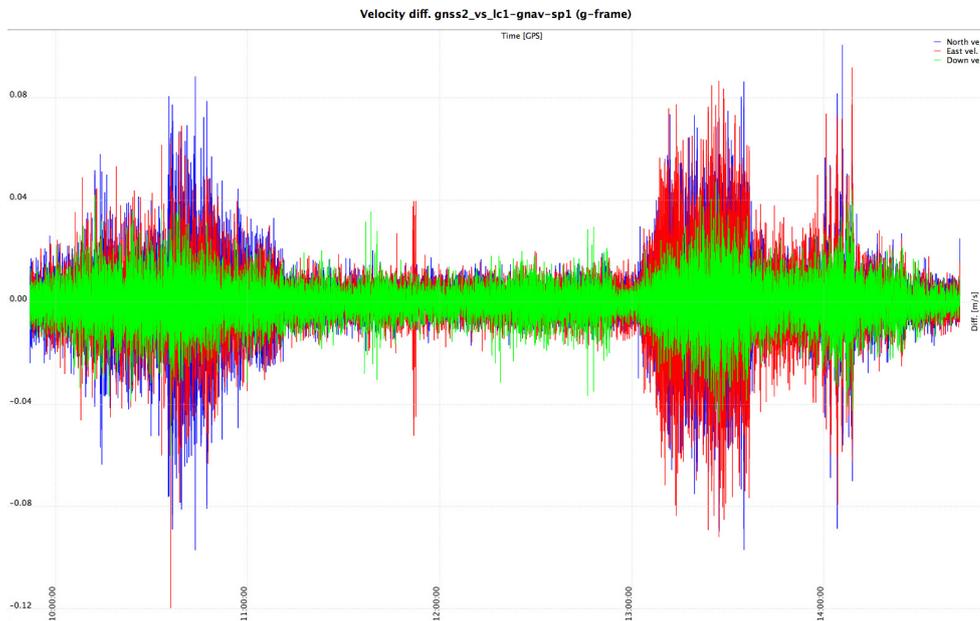


Figure 3: Velocity differences with respect to the ambiguity-fixed reference solution.

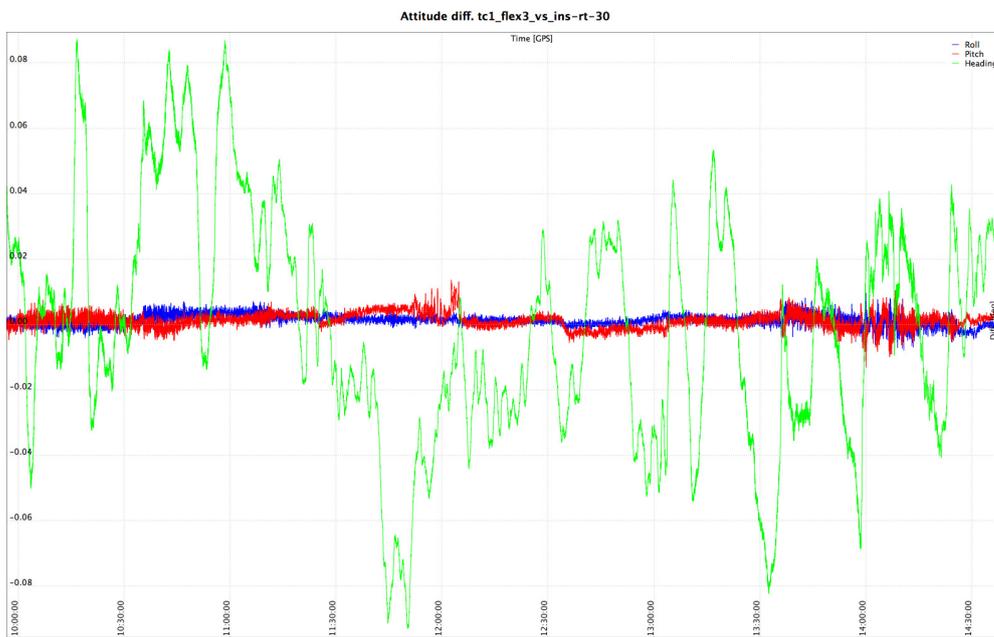


Figure 4: Attitude differences with respect to the reference solution.

## 7 Summary and Conclusion

It is a highly challenging task to obtain a reference trajectory with sub-centimeter accuracy. Using differential GNSS methods and a relatively short baseline, the TerraPOS PPP solution has been shown to be consistent at the level of a few centimeters. The reference trajectory has been processed with completely independent data (different antennas and receivers, different software). It is therefore evident that post-processed PPP really has centimeter-level accuracy potential. Positions and velocities from a tightly coupled PPP/INS solution can be expected to be at least as good as the standalone PPP solution, but a reference trajectory of sufficient accuracy is currently not available for this verification. It must also be taken into account that small errors in horizontal position differences are due to lever-arm compensation. The differences have been computed by comparing two different antennas, and attitude is then required. The heading used in this compensation is only accurate to roughly 0.2 degrees (a single-antenna INS solution), corresponding to approx. 10 mm RMS for a 2.5 m baseline.

The reference attitude is of sufficient accuracy to serve as truth in the comparison. Roll and pitch estimates from the TerraPOS solution are consistent with the reference at the level of 0.003° RMS. At this level, imperfect knowledge of the local deflection of the vertical will limit further improvements. The heading is consistent with the reference at the level of 0.035° RMS. As can be seen from the plots, the heading error changes slowly with time. This is likely to be due to the combined multipath signal of the two antennas. The error corresponds to less than 4 mm error for the 2.5 m baseline.

Overall, post-processing of Seapath 330+ data has yielded very favorable results when compared to the stated real-time accuracy of the system. Using positions, velocities and attitude from one unified processing ensures optimality and maximum consistency of the results.

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