Validering av GOCE satellittprodukter for gravimetrisk gradiometri: Forskningsmuligheter

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Michal Šprlák: Validation of GOCE Satellite Gravity Gradiometry Products: Research Opportunities

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Satellite gravity gradiometry has been successfully realized onboard the GOCE (Gravity field and steady-state Ocean Circulation Explorer) satellite. This mission provides homogenous and almost global gravity field information based on observations by a gradiometer. The gradiometer measures electric voltages to record test mass accelerations due to the Earth's gravity field. Transformation in several steps is needed to achieve the so called GOCE products. To ensure correctness of the transformation, the GOCE products have to be validated by independent knowledge about the Earth's gravity field. Several methods have been proposed to validate the GOCE products.

In this contribution, we firstly present fundamentals of GOCE to the readers who are unfamiliar with SGG. Secondly, spherical harmonic synthesis, upward continuation and downward continuation are considered for validation of GOCE products. A mathematical model of each method is proposed and possible challenges for research are discussed in the context of the Nova-GOCE (Norwegian validation and exploitation of GOCE gravity data) project.

Keywords: Calibration, Validation, Upward continuation, Downward continuation, Spherical harmonics

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Introduction

The gravitational attraction of the Earth is one of the most mysterious natural phenomena. It is a consequence of the gravity field present inside and in the neighborhood of our planet. We usually use the gravity field as a mirror to understand other natural phenomena such as the behavior of ocean currents, the inner structure of the Earth, sea level rise or ice melting. Engineering practice has also shown that one has to deal with the gravity field when building large-scale constructions such as bridges, dams, highways or tunnels.

Exploration of the Earth's gravity field is a non-trivial problem based on measurements of its properties. For this purpose traditional terrestrial and the novel satellite techniques have been applied. Employing methods of terrestrial gravimetry, magnitudes of gravity have been observed using pendulum devices, relative spring gravimeters, and free fall absolute gravimeters over a time span of several decades. Over inaccessible areas such as mountains, large forests, and oceans, the principles of airborne and marine gravimetry have been used. In spite of enormous personal effort and scientific merit, terrestrial gravimetry is characterized by particular variability in accuracy as well as irregular density and coverage of measurements. In addition, terrestrial gravity data are often related to different height and positioning systems. Their contribution to understanding global natural phenomena is therefore restricted.

To achieve global information about the Earth's gravity field with homogenous distribution and accuracy, satellite gravity missions were implemented. Several concepts for satellite gravity missions have been proposed. Probably due to tremendous technological scientific challenges and the need for extensive financial resources, only three of them have been realized successfully: CHAMP (CHAllenging Mini-Satellite Payload), GRACE (Gravity Recovery and Climate Experiment) and GOCE (Gravity field and steady-state Ocean Circulation Explorer).

Nowadays, GOCE (ESA, 1999) satellite mission deserves special attention. Using the concept of Satellite Gravity Gradiometry

(SGG) it extends our knowledge of the Earth's gravity field as surveyed by the two previous satellite missions. The gradiometer carried onboard the GOCE satellite is a sophisticated device. It is composed of 6 accelerometers, each measuring electric voltages. Electric voltages do not express the properties of the Earth's gravity field. Transformation into the so-called GOCE products, e.g. accelerations or gravitational tensor components, is performed. Based on the principle "Trust but verify" validation of the GOCE products is a crucial aspect. Its purpose is to ensure that measurement, processing and calibration phases have been performed correctly. In other words, we verify the correctness of the transformation from the original electric voltages into the GOCE products which are subsequently available to a broader scientific community.

The aim of this contribution is twofold. Firstly, we present some of the basic facts of SGG for the readers who are unfamiliar with the topic. For those familiar with SGG and those interested in advanced topics on SGG, references at the end of this paper and the special issue on GOCE satellite mission recently published in Journal of Geodesy are recommended. Secondly, we discuss research opportunities related to validation of GOCE SGG products. This is an important part of the Norwegian GOCE initiative (Haagmans and Pettersen, 2001) and one of the main goals of the Nova-GOCE project (Norwegian validation and exploitation of GOCE gravity data). We note that the research opportunities discussed in this paper were formulated at an early stage of the Nova-GOCE project (in second half of 2010). Since then, several conferences have been devoted to SGG. In addition, our knowledge of existing studies on GOCE validation has improved. Therefore, the list of references may not be considered as complete.

We have divided the paper into the following sections. In section 1, fundamentals of the GOCE satellite mission are introduced. Basic facts and measurements, objectives and applications, data levels and products and the Nova-GOCE project are briefly described. In section 2 we define validation and its purpose. Three different methods, i.e. spherical harmonic synthesis, upward and downward continuation are then introduced from the mathematical and geometrical point of view. Research opportunities for each of the methods are discussed. Important points from this paper are summarized in the conclusions.

1. Fundamentals of the GOCE satellite mission

SGG (Freeden and Schreiner, 2010; Rummel, 2010) may be directly defined as measuring the second derivatives of the gravitational potential at satellite altitude. They form the so-called gravitational tensor composed of 9 components. Since the gravitational field is non-rotational and continuous, the gravitational tensor is symmetric and traceless, and thus it is fully given by 5 components only (Rummel and van Gelderen, 1992). Another important property of the gravitational tensor is that compared to lower derivatives of the gravitational potential, the tensor components are much more sensitive to higher frequency features of the Earth's gravity field (Rummel and van Gelderen, 1995). For this reason, the concept of the SGG has been studied extensively and has only recently been realized with the launch of the GOCE satellite mission.

1.1 Basic facts and important measurements

The GOCE satellite mission belongs to the core Earth explorer missions of the European Space Agency (ESA, 1999) and is an important part of ESA's Living Planet Programme (ESA, 1998). The duration of the mission was originally planned to be 20 months. After its launch on March 17, 2009, low solar activity led to significant savings of fuel consumption. An extension of the mission has been approved until the end of 2012.

To obtain the gravity field with high resolution and global coverage, the orbit of the octagonally shaped spacecraft with wings aside, see Fig. 1, had to be designed properly. A very low altitude of approximately 250 km above the Earth surface enables recovery of features of the gravity field with a resolution of 100 km. The inclination of the satellite orbit with respect to the equator was chosen to be 96.7°. Obviously, with this inclination global data coverage is not complete, with data gaps in polar areas. On the other hand, the inclination allows a sun-synchronous orbit (with almost constant sun illumination). This is important due to the energy needs of the spacecraft.

Several important instruments have been placed onboard the GOCE satellite. Satellite-to-Satellite Tracking (SST) data are compiled through the GNSS (Global Navigation Satellite Systems) receiver and a special dual frequency antenna. SGG data are registered by a gradiometer which is composed of 6 accelerometers. In addition, the orientation and angular velocity of the satellite is controlled by three star trackers. The satellite itself is affected by gravitational and non-gravitational forces. To reduce non-gravitational forces the so called drag-free control system has been used (Canuto et al., 2003). For more information on GOCE orbit constellation and measurement principles, see (Drinkwater et al., 2007) and [1, 2].

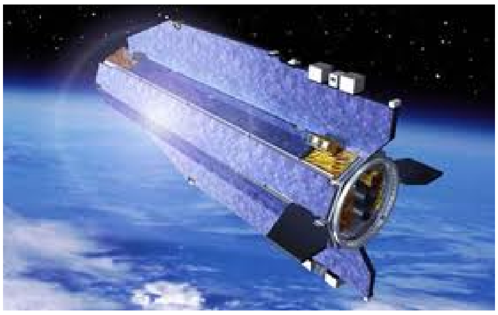

Fig. 1: GOCE satellite on orbit

1.2 Objectives and applications

GOCE maps the gravity field of the Earth with almost global coverage and homogenous accuracy. Its primary objectives are (Drinkwater et al., 2003):

– determination of a global gravity field (in the form of gravity anomalies or gravity disturbances) with an accuracy of 1 mGal,
– determination of a global geoid model with an accuracy of 1 cm,

with corresponding accuracy at a spatial resolution of 100 km. The main objectives are particularly relevant for exploitation in the following multidisciplinary applications:

– realization of the global height system in geodesy,
– better understanding of the structure and geodynamical processes inside the Earth's body in geophysics and geodynamics,
– understanding ocean circulation in oceanography and climate-related sciences,
– mass transport and ice thickness in glaciology.

1.3 Data levels and products

In principle, the only role of the spacecraft is to perform observations and send all information to the ground. Subsequently, additional operations and processing are carried out by the so called processing facilities. All GOCE data are organized into a hierarchical structure forming three levels of products (Drinkwater et al., 2003).

At the first level, time-ordered data produced by the satellite platform and instruments form the so called Level 0 products. These are composed of (ESA, 2010):

– satellite and instrument internal data,
– output of the 6 accelerometers,
– SST and star trackers' data.

Level 0 products are subsequently calibrated, localized on the satellite orbit and converted into engineering units by the Instrument Processing Facility (IPF). The resulting Level 1b products become available under the following abbreviations:

– SST_NOM_1b (SST-related products: carrier phase and pseudorange measurements, outlier detection, bias estimation, covariance matrix, etc.),
– SST_RIN_1b (SST data in the conventional RINEX format),
– EGG_NOM_1b[3] (SGG-related products: electric voltages, accelerations, angular

1. http://earth.esa.int/GOCE/
2. http://www.esa.int/esaLP/LPgoce.html
3. First three letters of the abbreviation EGG stand for Electrostatic Gravity Gradiometer

accelerations, gravitational tensor components, etc.).

At the High-level Processing Facility (HPF; Bouman et al., 2009) Level 1b products are then searched for outliers, corrected for time-variable effects and processed to obtain the following Level 2 products (EGG-C, 2010a)[4]:

- EGG_NOM_2_ (calibrated and corrected gravitational tensor components in the gradiometer reference frame),
- EGG_TRF_2_ (calibrated gravity gradients in the local north-oriented reference frame),
- SST_PSO_2_ (reduced-dynamic and kinematic orbits),
- EGM_GOC_2_ (GOCE derived spherical harmonic coefficients and functionals of the gravitational potential evaluated in a regular grid),
- EGM_GVC_2_ (full variance-covariance matrix of the spherical harmonic coefficients),
- SST_AUX_2_ (time-variable spherical harmonic coefficients).

Level 1b and Level 2 products are available for further exploitation by the geo-scientific community. Authorized users can download these products in the form of xml files from the GOCE Virtual Online Archive[5]. Alternatively, one may also order Level 1b and Level 2 products through a client-server application called EOLI-SA, see (VEGA, 2010) and[6]. At the highest level, advanced and up-to-date Earth-related models are produced by the scientific community. These represent the so-called Level 3 products based on, for example, the terrestrial, CHAMP and GRACE data, and the GOCE Level 1b and Level 2 products.

1.4 Project Nova-GOCE

Validation and exploitation of the GOCE products through national or international projects worldwide are further challenges for the scientific community. Nova-GOCE represents a Norwegian attempt to validate and exploit the GOCE products. It is funded for three years (2010–2012) by the Space Research Programme of the Norwegian Research Council and is based at the Department of Mathematical Sciences and Technology, Norwegian University of Life Sciences. Collaboration with the National Mapping Authority in Norway as well as international institutions such as Institute of Astronomical and Physical Geodesy, Bavarian Academy of Sciences and German Geodetic Research Institute in Munich is planned. The three main goals of the project are analysis of the existing gravity database in Norway, validation of GOCE products, and gravity field recovery for Nordic countries. Section 2 of this contribution deals with the possibilities of validation of the GOCE products. More information on the Nova-GOCE project can be found in (Šprlák et al., 2011).

2. Validation of GOCE SGG products

The GOCE gradiometer is composed of 6 accelerometers. Pairs of accelerometers are aligned along the axes of the Gradiometer Reference Frame (GRF; EGG-C, 2010b) respecting the orbital motion of the satellite, see Fig. 2. Originally, electric voltages induced by the changing equilibrium of test masses are measured by the accelerometers. The electric voltages are indirectly related to the Earth's gravity field, but scale factors are required to transform electric voltages into accelerations. Common mode accelerations, i.e. addition of accelerations along the same coordinate axis, are used by the drag-free control system of the spacecraft to compensate for non-gravitational forces. From differential mode accelerations, i.e. subtraction of accelerations observed along the same coordinate axis, gravitational tensor components in the GRF are obtained. Transformation of the gravitational tensor components from the GRF into the Earth-related local north-oriented reference frame (LNORF) is performed, see Fig. 2. Moreover, gravitational tensor components in the GRF are used to evaluate spherical harmonic coefficients of the GOCE-derived global gravity models (GGMs).

4. Abbreviation EGG-C means European GOCE Gravity Consortium
5. http://eo-virtual-archive1.esa.int/Index.html
6. http://earth.esa.int/EOLi/EOLi.html

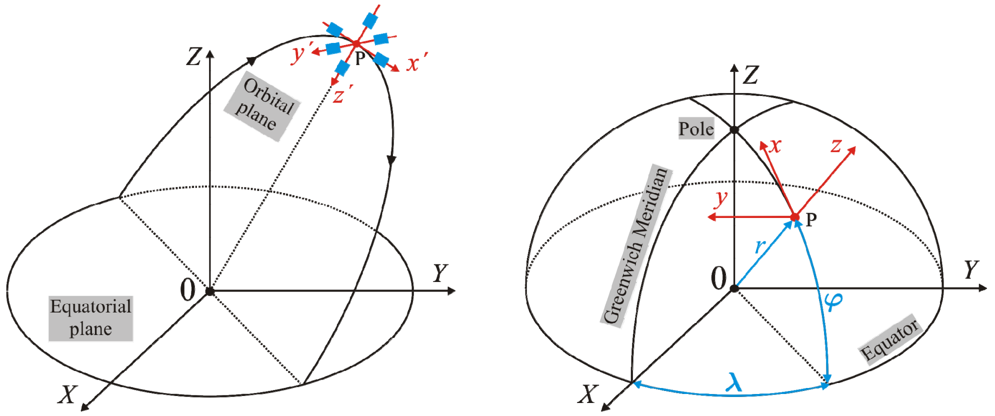

Fig. 2: Left: Relation of the geocentric Cartesian (X, Y, Z) and the gradiometer (x´, y´, z´) reference frame (accelerometers depicted by blue boxes). Right: Relations of the Earth-fixed reference frames: (X, Y, Z) – geocentric Cartesian, (r, φ, λ) – spherical geocentric, (x, y, z) – local-north oriented.

The process of obtaining GOCE products suitable for geo-scientific applications from the original observations is relatively complex. It is therefore necessary to verify this process and the final products qualitatively and quantitatively. The purpose of validation is defined formally as follows (Koop et al., 2001): *Application of methods to compare data products derived from measurements with existing independent data or knowledge in order to assess the quality of the data products and to make sure that the measurement process, error estimation and calibration have been performed correctly.* One may also encounter with the term calibration which has a different meaning. According to Koop et al. (2001) calibration is *a procedure to determine parameters and the application of these parameters to the instrument read-outs in order to obtain quantities in the required physical units and dimensions and with the required and known accuracy.* Given the definitions above, it is important to realize that while the calibration is related to the instruments placed onboard GOCE, validation is a test of the GOCE products.

Several approaches have been proposed for validation of the GOCE products (Haagmans et al., 2003; Pail, 2003; Zielińsky and Petrovskaya, 2003; Bouman et al., 2005; Kern and Haagmans, 2005; Visser, 2007; Wolf, 2007; Eshagh, 2009b). Using the methods proposed, one may verify a broad spectrum of GOCE products. For the sake of brevity, we restrict ourselves here to validation of the GOCE Level 2 SGG products EGG_NOM_2_ and EGG_TRF_2_. Geometrically, such a validation may be interpreted as follows, see Fig. 3. Suppose gravity anomalies Δg or gravity disturbances δg to be available at the Earth surface. Moreover, several GGMs defining the Earth's gravity field also exist. At the satellite orbit, gravitational tensor components V_{ij} are deter-

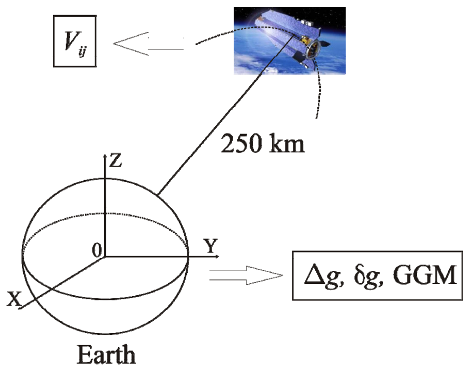

Fig. 3: Formulation of the validation problem

mined by the GOCE satellite mission. For validation purposes, a mathematical apparatus is used to relate existing gravity data at the Earth surface to the gravitational tensor components at the GOCE altitude. Spherical harmonic synthesis, upward continuation and downward continuation may be used. The potential applications and restrictions of these methods in the validation process are discussed in the next subsections.

2.1 Spherical harmonic synthesis

Spherical harmonic synthesis is widely used in global and regional gravity field modeling. This approach is based on expansion of the gravitational potential in terms of spherical harmonics (Heiskanen and Moritz, 1967). Similarly, other functionals of the gravitational potential (e.g. geoid undulation, deflections of the vertical, tensor components) may be expressed. For GOCE validation purposes the following mathematical model is relevant:

$$V_{ij}(r,\Omega) = \frac{GM}{a^3}\sum_{n=0}^{M_{\max}}\left(\frac{a}{r}\right)^{n+3}\sum_{m=0}^{n} C_{nm}\, Y_{nm}^{ij}(\Omega), \quad i,j = x,y,z \qquad (1)$$

where $[r, \Omega=(\phi, \lambda)]$ is the triplet of the spherical coordinates, i.e. spherical radius, spherical latitude and longitude, V_{ij} are the gravitational tensor components in the LNORF, GM is the product of the Newtonian gravitational constant and the mass of the Earth including oceans and atmosphere, a is the semi-major axis of the reference ellipsoid, M_{max} is the maximum degree and order of the expansion, C_{nm} are the spherical harmonic coefficients of degree n and order m, and Y_{nm}^{ij} are the linear combinations of the spherical harmonics and their derivatives corresponding to the gravitational tensor components. From the computational point of view, evaluation of the gravitational tensor components at satellite altitude is performed by a simple double summation of the spherical harmonic coefficients multiplied by the linear combinations of the spherical harmonics and their derivatives. The set of spherical harmonic coefficients forms a GGM. Database of such models is available at the International Centre for Global Earth Models (ICGEM, see http://icgem.gfz-potsdam.de/ICGEM/). In practical computations, the linear combinations of the spherical harmonics and their derivatives have to be computed numerically. However, conventional expressions develop singularity towards the poles. When one is interested in evaluating the gravitational tensor components close to or directly at the poles, alternative non-singular expressions derived in (Petrovskaya and Vershkov, 2006; 2008; Eshagh, 2009a) are recommended.

Based on the spherical harmonic synthesis approach, validation of the EGG_TRF_2_ product is straightforward. Before validation one has to consider that the EGG_TRF_2_ product has been obtained by a transformation of the EGG_NOM_2_ gravitational tensor components from the GRF into the LNORF. However, the transformation is highly affected by the accuracy of all gravitational tensor components. For this reason, less accurate gravitational tensor components are replaced by model values computed by an a-priori GGM (EGG-C, 2010a; Fuchs and Bouman, 2011). After the replacement, point-wise transformation of the gravitational tensor components is performed. Obviously, validation of the EGG_TRF_2_ product is affected not only by the measurement and calibration process, but also by the transformation and the accuracy of the a-priori GGM.

It is probably more reasonable to validate the EGG_NOM_2_ product, i.e. gravitational tensor components in the GRF. First, gravitational tensor components have to be evaluated according to Eq. (1) using a proper GGM. In contrast to the GOCE observations, we may expect the same order of accuracy for all gravitational tensor components. Subsequent transformation from the LNORF into GRF is not as critical as its inverse. Another aspect to consider is validation of the EGG_NOM_2_ product in a certain bandwidth since the accuracy of the tensor components is higher in the measurement bandwidth of the gradiometer in the range from 5 to 100 mHz. Filtering of the evaluated and transformed gravitational tensor components with sufficient accuracy is therefore required.

2.2 Upward continuation

Validation by upward continuation may be performed above countries with sufficient coverage and homogenous quality of terrestrial gravity data. Terrestrial gravity data available on the ground is transformed into tensor components at the GOCE satellite altitude. Imaginary geometry of the gravity field (represented by equipotential surfaces) is smoother with increasing altitude; therefore, the upward continuation is also known as a smoothing procedure. This implies that upward continuation does not cause any numerical problems. From the mathematical point of view, one may solve this direct problem by least squares collocation or by integral transformations. In the following subsection, we restrict ourselves to the application to integral transformations. Readers interested in the least squares collocation used for the GOCE validation are referred to in, for example, (Wolf, 2007, chap. 5.2), and references therein.

The starting point in formulating upward continuation integral transformations is the well known Pizetti's (extended Stokes's) formula (Heiskanen and Moritz, 1967, Eq. 2-161):

$$T(r,\Omega) = \frac{R}{4\pi}\iint_{\Omega} S\left[r,R,\psi(\Omega,\Omega')\right] \times \Delta g(R,\Omega')\,d\Omega' \qquad (2)$$

The disturbing potential T is evaluated by a surface integral over a sphere Ω with radius R. Inside the surface integral, the extended Stokes' function S and the gravity anomalies Δg are multiplied. While the gravity anomalies are given by observations, values of the extended Stokes' function are numerically computed based on radii r, R and the spherical distance ψ between the computational point and the integration element, see Fig. 4. Note that the disturbing potential is defined as the difference between the actual gravitational potential (generated by the real Earth) and a normal gravitational potential (generated by a mathematical body, such as rotating ellipsoid).

Disturbing tensor components T_{ij} in the LNORF can be obtained by differentiating Eq. (2), (Kern and Haagmans, 2005; Wolf, 2007):

$$T_{ij}(r,\Omega) = \frac{R}{4\pi}\iint_{\Omega} S_{ij}\left[r,R,\psi(\Omega,\Omega'),\alpha(\Omega,\Omega')\right] \times \Delta g(R,\Omega')\,d\Omega', \quad i,j = x,y,z \qquad (3)$$

where S_{ij} are the integral kernels depending not only on r, R and ψ, but for certain tensor components also depending on the spherical azimuth α, see Fig. 4. In analogy to the previous note, the disturbing potential tensor is

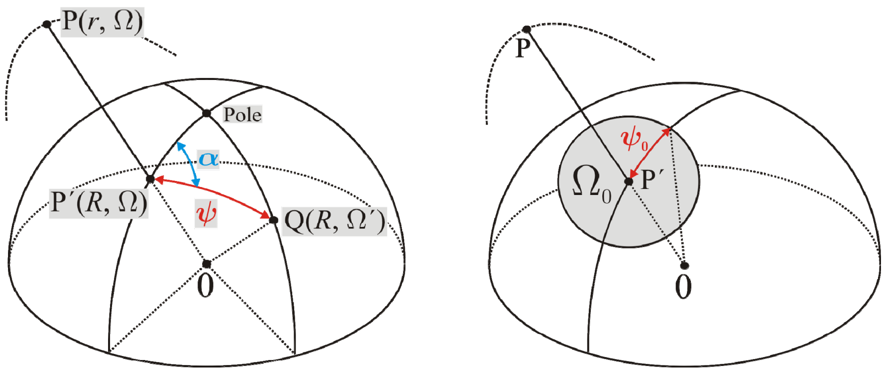

Fig. 4: Left: Geometry of the integral transformations. P(r, Ω) is the evaluation point at the GOCE satellite orbit, P´(R, Ω) being its projection on a reference sphere, Q(R, Ω´) is the integration element. Right: Limited area of integration. Symbol Ω substitutes a pair of the spherical latitude φ and the spherical longitude λ, i.e. Ω = (φ, λ). Ω_0 is an area on a sphere within a spherical cap ψ_0.

given by subtraction of the actual and the normal gravitational tensors. This simple relationship allows connection between the gravitational tensor observed by the GOCE satellite mission and the disturbing tensor evaluated by Eq. (3).

Theoretically, knowledge of the gravity anomalies over the whole Earth is required in both Eqs. (2) and (3). In practice, the surface integral is decomposed into near and distant zones. The effect of the near zones is evaluated by integration of the gravity anomalies in the area Ω_0 within a limited spherical cap ψ_0, see Fig. 4. The effect of the distant zones is simply expanded in the series of the spherical harmonics. Considering this in Eq. (3), practical disturbing tensor estimators may be formulated as follows (Eshagh, 2009b):

$$\begin{aligned}&\hat{T}_{ij}(r,\Omega)\\&=\frac{R}{4\pi}\iint\limits_{\Omega_0} S_{ij}\left[r,R,\psi(\Omega,\Omega'),\alpha(\Omega,\Omega')\right]\\&\qquad\times\Delta g(R,\Omega')\,\mathrm{d}\Omega' \qquad (4)\\&+\frac{R}{2}\sum_{n=2}^{M_{\max}} Q_n^{ij}(r,R,\psi_0)\,\Delta g_n^{ij}(\Omega),\\&\quad i,j=x,y,z\end{aligned}$$

where Q_n^{ij} are the truncation error coefficients and Δg_n^{ij} is the n-th Laplace surface spherical harmonics which corresponds to a disturbing tensor component. The truncation error coefficients, i.e. weights in the spectral domain, are evaluated numerically using recurrence formulae (Thalhammer, 1994). The Laplace surface spherical harmonics are computed by the series of spherical harmonics and a GGM. Alternative disturbing tensor estimators may be found by applying the Remove-Compute-Restore (RCR) technique (Wolf, 2007). In this approach, decomposition of the surface integral in spatial and spectral domain is performed. However from a mathematical point of view, pure spatial decomposition in Eq. (4) and the RCR technique are equivalent. Let us also note that the original upward continuation in Eq. (3), assuming gravity anomalies only, has changed into the combination of the gravity anomalies and a GGM in Eq. (4).

Many practical aspects need to be considered when evaluating and applying disturbing tensor estimators (4). Some of them are well known from the problem of geoid determination. Other aspects are challenging for the scientific community as they seem to be new. At first, we do not know exactly how errors of the gravity anomalies and of the spherical harmonic coefficients affect the disturbing tensor components evaluated at satellite altitude. It is therefore of particular interest to analyze the error propagation in Eq. (4). Such an analysis will help to determine optimal values of input parameters, e.g. size of the integration cap and the maximum degree of the spherical harmonic coefficients.

One challenging task relates to the properties of the integral kernels. Original integral kernels are often modified in practice to improve the convergence rate of the distant zones and to improve filtering properties. Many strategies for modifying integral kernels have already been proposed for geoid determination and some generalization attempts have been made (Sjöberg, 2003; Šprlák, 2010). Regarding the disturbing tensor components, only a few examples of modifying integral kernels have been discussed (Wolf, 2007; Eshagh, 2009b). A comprehensive study involving the existing modification approaches is therefore required.Thus, we may understand the real benefit of the proposed ideas and their applicability not only for geoid determination but also for determination of the disturbing tensor components or even for higher order derivative functionals of the disturbing potential.

Standard practice in geoid determination requires that atmospheric, topographic and ellipsoidal corrections are included in the computation. The same corrections have to be included when accuracy of the disturbing tensor components at the level of 1mE ($1\ \mathrm{E} = 10^{-9}\ \mathrm{s}^{-2}$) is expected. Eshagh and Sjöberg (2009) analyzed atmospheric and topographic corrections for the disturbing tensor components. They found the long-wavelength effect caused by the above mentioned corrections on the level of several tens of mE. The effect implied by the spherical approximations used in Pizzeti's formula (2), i.e. ellipsoidal correction, has not been discussed for the case of the disturbing tensor components. Alternatively, to avoid ellipsoidal correction, one may obtain disturbing tensor components from an ellipsoidal extension of Pizzeti's formula (Martinec and Grafarend, 1997). Both problems, i.e. analysis of the ellipsoidal correction and deriving disturbing tensor components from the ellipsoidal Pizetti's formula, are also a challenge for the scientific community.

2.3 Downward continuation

Except for the direct evaluation of Eqs. (3) or (4), one may solve an inverse problem when the disturbing tensor components at the satellite altitude are known and the gravity anomalies on the ground are sought. In accordance with the direction (from the satellite level towards the ground), this procedure is called downward continuation (Reed, 1973; Xu, 1992; Janák et al., 2009). Downward continuation is of particular interest in regional geoid modeling when gravity anomalies over neighboring countries or poorly surveyed areas are required. Supposing that independent gravity data exist on the ground, we can use the downward continuation for GOCE validation purposes.

Geometric interpretation of the downward continuation indicates its possible difficulties. Imaginary geometry of the gravity field represented by equipotential surfaces is smoother at higher altitudes, i.e. at the level of the observed disturbing tensor components. However, more complicated features of the equipotential surfaces are to be recovered on the ground making the downward continuation numerically unstable. Indeed, numerical experiments prove that we are able to obtain gravity anomalies on the ground with an accuracy of several mGal (Pail, 2003; Janák et al., 2009). However, such a level of accuracy is insufficient in our validation purposes, as we suppose the accuracy of the terrestrial gravity anomalies over well surveyed areas to be higher by one order of magnitude.

Let us now illustrate one possible application of downward continuation in the context of the GOCE satellite mission. Suppose a domain Ω' in a three-dimensional space bounded from above by the GOCE satellite orbit Γ_{GOCE}, from below by the Earth surface Γ_1 and on the sides by the boundaries $\Gamma_2,\ldots,\Gamma_5$, see Fig 5. Suppose the Laplace differential equation is valid in the domain Ω'. Suppose the disturbing tensor components on Γ_{GOCE} and the gravity anomalies on $\Gamma_1,\ldots,\Gamma_5$ are given. We can then formulate the following Terrestrial Gravity-Satellite Gravity Gradiometry Boundary Value Problem (TG-SGG BVP):

$$\nabla^2 T = 0 \text{ in } \Omega' \tag{5}$$

$$\Delta g = -\frac{\partial T}{\partial r} - \frac{2}{r} T \text{ on } \Gamma_k, k = 1,2,\ldots,5 \tag{6}$$

$$T_{ij} = t \text{ on } \Gamma_{\mathrm{GOCE}} \tag{7}$$

In a limiting case, when we consider gravity anomalies distributed globally, the side boundaries $\Gamma_2,\ldots,\Gamma_5$ disappear (Holota and Nesvatba, 2007).

It is known from variational calculus that the highest possible order of the derivative of the unknown function in a boundary condition is at most one half the order of a differential equation (Rektorys, 1980). In the case of the TG-SGG BVP, the derivative of the disturbing potential in the boundary condition (7) is of the same order as of the Laplace differential equation (5). This restriction may be overcome by applying the downward continuation, inverse of Eq. (3), when the second derivative of the disturbing tensor is continued and transformed into gravity anomalies. Then the TG-SGG BVP is transformed into the following BVP:

$$\nabla^2 T = 0 \text{ in } \Omega'' \tag{8}$$

$$\Delta g = -\frac{\partial T}{\partial r} - \frac{2}{r} T \text{ on } \Gamma_k', k = 1,2,\ldots,6 \tag{9}$$

We note that the transformation of the boundary condition implies additional transformation of the original domain Ω' into a domain Ω'', see Fig. 5. This new domain Ω'' is bounded from above by the boundary $\Gamma_6' \neq \Gamma_{GOCE}$, from below by the Earth surface $\Gamma_1 = \Gamma_1'$ and on the sides by the boundaries $\Gamma_2' \neq \Gamma_2,\ldots,\Gamma_5' \neq \Gamma_5$. Because we propose downward continuation of the disturbing tensor components into the gravity anomalies, upper boundary $\Gamma_6{'}$ may be chosen arbitrarily. A limiting factor is the precision of the downward continuation. Therefore an optimal height of the boundary $\Gamma_6{'}$ has to be determined (Janák et al., 2011).

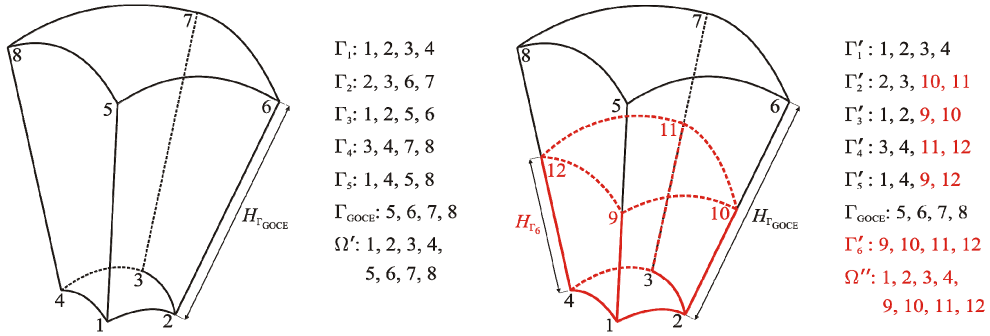

Fig. 5: Left: Geometry of a solution domain for the TG-SGG BVP. Right: Change of the upper boundary in the TG-SGG BVP.

Conclusions

SGG realized onboard the GOCE satellite represents a sophisticated observational technique. Initial electric voltages induced by the test masses inside accelerometers are transformed into accelerations and further into gravitational tensor components. Such a transformation procedure requires validation by independent knowledge of the Earth's gravity field. Three mathematical models, i.e. spherical harmonic synthesis, upward and downward continuation, widely used in geodesy, are suitable for the validation.

Spherical harmonic synthesis allows evaluation of the gravitational tensor components from a set of spherical harmonic coefficients. The EGG_TRF_2_ product can be validated directly by the spherical harmonic synthesis. When doing so, one has to take into account a-priori information and all transformations involved within the EGG_TRF_2_ product. It is probably more reasonable to validate the EGG_NOM_2_ product. Gravitational tensor components evaluated by spherical harmonic synthesis have to be transformed into the GRF. Subsequently, proper filtering is required to validate gravitational tensor components in a frequency spectrum which corresponds to the measurement bandwidth of the gradiometer.

Over well surveyed areas upward continuation may be performed by integral transformations. Due to limited availability of the global set of terrestrial gravity anomalies, we have to formulate disturbing tensor estimators properly. Several practical aspects of evaluating and applying disturbing tensor estimators have to be investigated. Of particular interest for the scientific community are error propagation, modifications of integral kernels and the effect of the ellipsoidal correction. Alternatively, we may derive disturbing tensor components from the ellipsoidal extension of Pizetti's formula.

Downward continuation of the disturbing tensor into the gravity anomalies may theoretically be exploited for validation purposes as well. However, this is a numerically unstable procedure which allows recovery of the gravity anomalies with an accuracy of several mGal. Its practical application for the GOCE validation purposes is therefore restricted. We have shown one possible application of the downward continuation in the context of the GOCE mission. Requiring TG-SGG BVP to be solvable, downward continuation based on inverse of Eq. (3) may be applied to weaken the order of the derivative of the boundary conditions.

The process of validation is not only difficult from a theoretical point of view. When we consider arbitrary GOCE products or the terrestrial gravity anomalies, we have to deal with a huge number of data points (from tens of thousands up to millions). An important task and integral part of validation is therefore development of effective software. In addition to algorithms implementing mathematical methods for the validation, one needs supporting routines for handling GOCE products in XML format or for computing the normal gravitational tensor. We strongly believe some of the theoretical challenges described above, as well as development of software, will be successfully addressed in the context of the Nova-GOCE project.

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