Et gravimetritestfelt for instrumentvalidering og utdanningsformål

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Vegard Ophaug et al.: A gravity test field for instrument validation and educational purposes

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Physical geodesy is a major component of the graduate curriculum for a MSc in geodesy at the Norwegian University of Environmental and Life Sciences (UMB) at Ås. It is also a significant component of current geodetic research at UMB. Terrestrial observations of the magnitude of gravity and its temporal variations have been made at Scandinavian stations since 2004 with a state-of-the-art free-fall absolute gravimeter of type FG5. This instrument has also been exploited for educational purposes in the graduate course in geodesy. In addition a LaCoste and Romberg (LCR) relative gravimeter is available, primarily for determination of vertical gravity gradients at FG-5 observing stations. Here we describe the initial establishment of a gravity test field at the university campus in Ås using both absolute and relative gravimetry. Its first application is to verify the scaling factor of the digitized LCR instrument over a small measurement range. Further extensions and tests will be conducted as part of future course work.

Key words: Gravity test field, instrument validation

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Introduction

Gravitational acceleration is a key observable describing the physics of the Earth. It is of fundamental importance to geodesy because the direction of the gravity vector is the only means to establish a physically meaningful definition of "up" and "down". Thus gravity is indispensible for the definition of vertical reference frames. It also gives insight into the density distribution of the Earth and, temporal gravity variations allow monitoring mass transport in the Earth system. Gravimetric observations are performed on the ground, on ships and airplanes, and by means of satellite techniques. A major component of geodetic research at the Norwegian University of Life Sciences (UMB) is concerned with observation and analysis of absolute gravity values in Fennoscandia considering spatial and temporal gravity variations (Breili 2009, Lysaker 2009, and Pettersen 2011). Besides this Šprlák et al. (2012) have collected data from ESA's satellite gradiometry mission GOCE (ESA, 1999) for external validation of GOCE products with terrestrial gravity data in Norway. Advanced instruments for terrestrial gravity observations are available at the Geomatics section of UMB. Since 2004 the section carries out its own observational program with the absolute gravimeter FG5-226. A relative gravimeter by LaCoste and Romberg (LCR G-761) has primarily been used for determination of vertical gravity gradients at FG5 observing sites. The two instruments are also exploited for educational purposes in UMB's graduate course in geodesy. Thus students get familiar with state-of-the-art observation techniques. In 2012 an initial gravity network was established at the university campus in Ås, with the intention of extending the student's project work to network adjustment and in future courses also to the analysis and geodetic exploitation of local gravity information. Combination of relative and absolute gravimetry also allows verifying the calibration parameters of the relative instrument, at least in the small measurement range covered by the local network. This is the first application of the newly established network on which we report in this contribution.

The gravity network is shown in Figure 1. It comprises two absolute gravity stations

(TF03 and NF01) which establish reference values of local acceleration of gravity to which the relative observations are tied. The topography allowed a height difference of 30 m between the two sites. The lower site (NF01) was established in the ground floor of a new building of the Norwegian Institute of Food, Fisheries and Aquaculture Research (Nofima). The higher site (TF03) is located at UMB's gravimetry laboratory in wing III of the Department of Mathematical Sciences and Technology building. Both sites are established on concrete floor. Four additional relative gravity stations (SH01, CI01, UR01 and IT01) were established, all of them in front of campus buildings. In order to ease access to NF01 for relative observations and to secure the absolute site against interior work in the Nofima building the eccenter point NF02 was established at the concrete entrance stairs to the east of the building. A detailed description of all observation sites is given in Ophaug (2012). In the following we provide an overview of the instrumentation, give details on the field work carried out in spring 2012, and show the results of the first network adjustment.

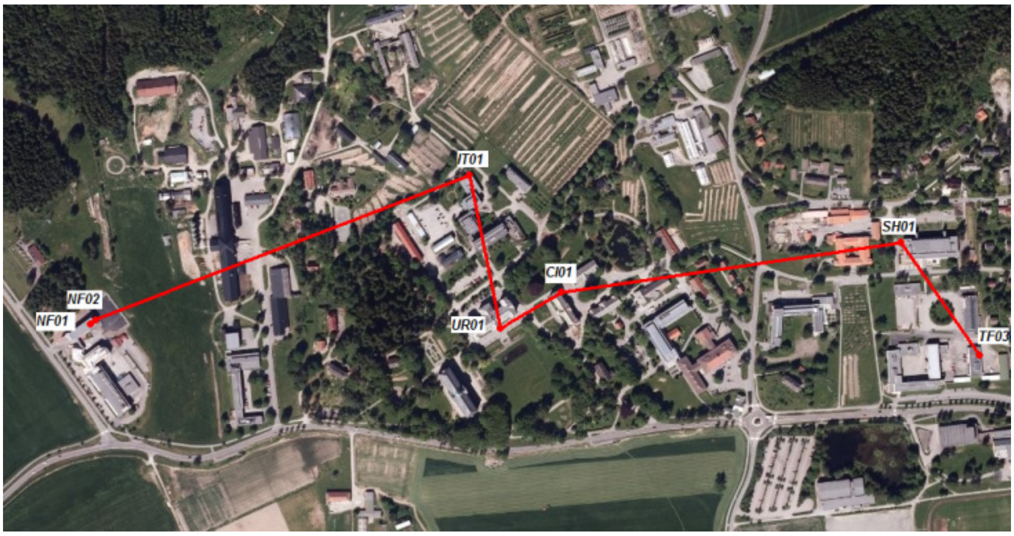


Figure 1: Gravity network at the UMB campus measured in 2012.

Instrumentation

An absolute gravimeter (FG5-226; http://www.microglacoste.com) was employed to establish the two reference sites TF03 and NF01. Absolute gravimeters provide the absolute value of gravity at an observing site from measurements of the fundamental quantities time and length. The FG5 instrument applies the principle of free-fall. Repeated observations of a free fall test mass are conducted and the final gravity value is computed from the average of all single drops. In our experiment 50 repeated drops were averaged to one set value and 24 sets were observed at hourly intervals. One assumes that errors of the tidal models employed for data reduction average to zero over an observation period of 24 hours. The test mass of the absolute gravimeter is dropped in a vacuum chamber which defines the reference height of the observation to be 120 cm above the floor. Reduction of the observed value to a reference marker on ground requires precise knowledge of the local vertical gravity gradient. Therefore a relative gravimeter is used to observe the change in gravity between the floor level and a platform about 1.4 m above the floor. The expected precision of FG5 observations is at the level of 10^{-9} of g or approximately 1–2 µGal.

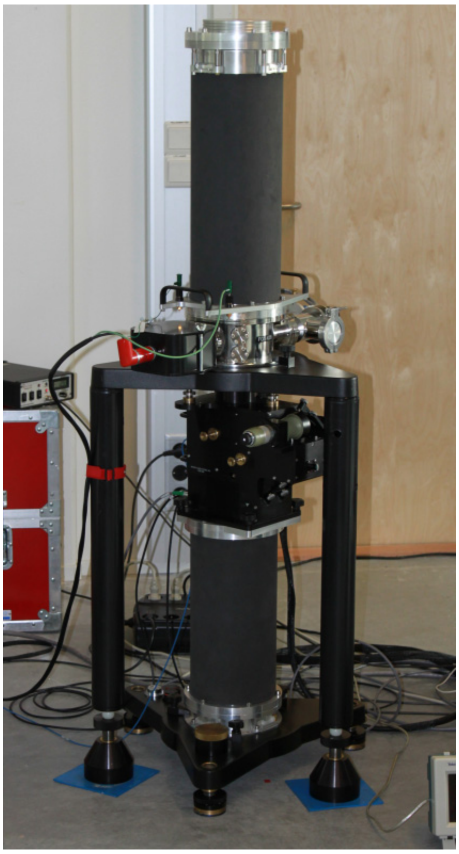


Figure 2: FG5-226 installed at NF01.

Figure 3: LCR G-761 set up on the stone stairways at CI01.

The LCR instrument only yields the difference in gravity between two stations. Relative gravimeters use a counterweight to maintain a test mass at equilibrium with gravity. Change of gravity in space or time is monitored by corresponding changes of the counterweight position, which is transformed to the unit of gravity by a calibration function. Our instrument, LCR G-761, is a digitized version with an electronic feedback system, i.e. the equilibrium position does not have to be set by the observer but is set automatically by the system within a certain measurement range. This way, also inexperienced observers can derive accurate readings thus making the instrument suitable for educational applications. The system automatically applies a scaling factor, thus providing gravity difference readings in units of mGal directly. Possible errors of the scaling factor can be found by external calibration, e.g. by comparison with absolute gravity values. Due to aging of the spring material and environmental disturbances (e.g. temperature changes in the sensor case or abrupt mechanical impulses arising during instrument transportation) the gravity difference reading changes with time (Timmen, 2010). This drift needs to be determined by repeated gravity station setups over one day and reduced by subsequent modeling. The expected precision of LCR-G observations is at the level of 10^{-8} of g or approximately 10–20 µGal. For further details on absolute and relative gravimeters we refer to Timmen (2010) and the references therein.

Field work

Observations were carried out in the spring semester 2012, between Sunday, March 18, and Wednesday, March 21. Due to time constrains not all FG5 sessions cover a full period of 24 hours. Three independent sessions were conducted at TF03 on three separate dates with observation periods of 24, 14 and 18 hours each. One 24 hours session was conducted at NF01.

Vertical gravity gradients were measured at NF01 and TF03 respectively at a time when FG5-226 was installed on the other of these two absolute sites. The gradient is derived from repeated observation of the gravity difference between ground level and a plate on top of a tripod at a distance of about 1.4 m above ground level (see Figure 4). At NF01 eleven repetitions were conducted, at TF03 only six. The gravity gradients were found to be –3.00 ± 0.11 µGal/cm at TF03 and –3.03 ± 0.07 µGal/cm at NF01 both of which are very close to the theoretical normal free-air gravity gradient of –3.086 µGal/cm. A number of previously measured values exist for TF03. When combined with the current value, the average gravity gradient for TF03 is –2.99 µGal/cm.

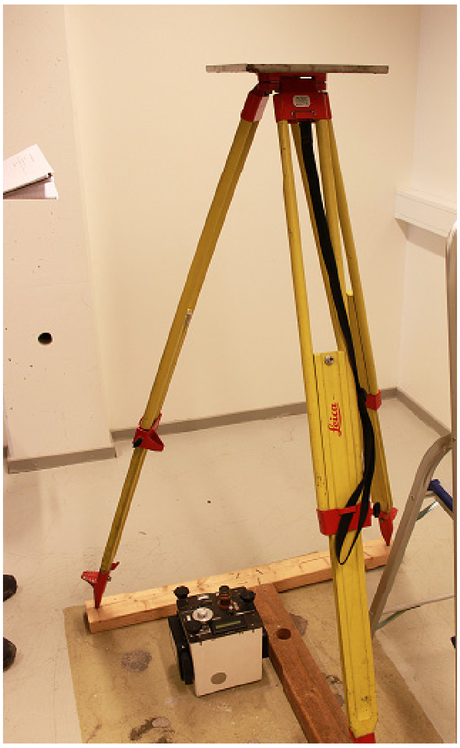


Figure 4: Tripod for vertical gravity gradient measurement with LCR G-761 at TF03.

Observation of gravity differences between all network stations were performed on one day with LCR G-761. The instrument was transported between the different stations by car. Because FG5-226 was installed at NF01, only the eccenter point NF02 could be accessed at that day. Before FG5 had been installed, the LCR instrument was used to transfer the floor gravity value to the outside eccenter NF02. The gravity difference between NF01 and NF02 was measured eight times and the average difference was found to be 1.6 ± 5.6 µGal. At the demonstrated precision of the relative gravimeter we consider this difference as insignificant. Thus we consider the ground value of NF02 equal to NF01. During network measurement the absolute sites were visited three times and the other stations twice. At two setups (first visit at TF03 and second visit at UR01) two sessions were conducted. Thus there are 16 relative observations available at the 6 network sites. This allows good control of the instrument drift and the repeatability of relative gravity observations.

Preprocessing and data reduction

Raw gravity observations contain various time variable components, several of which can be taken care of by model reductions. These are variations due to solid Earth and ocean tides, polar motion, loading effects and atmospheric mass movements (Timmen 2010). The major components of tidal effects are related to periods around 12 and 24 hours. The total gravity variations may reach almost 300 µGal and are therefore well above the precision of relative and absolute gravimeters. Thus proper reduction of the tidal gravity effect is necessary. The program ETGTAB originally created by H.-G. Wenzel (see http://www.bfo.geophys.uni-stuttgart.de/etgtab.html) is implemented in the FG5 processing software and was also used to provide tidal reductions for the LCR observations. Effects due to polar motion and atmospheric variations usually stay below ± 10 µGal and vary on longterm scales. Due to the short time interval between relative gravity readings (in our case the observing session lasted less than 3 hours) reductions are negligible for the LCR data. The absolute gravity values, however, need to be reduced considering that absolute observations might be repeated after long time spans, e.g. several years. In addition all gravity values are reduced to ground markers. In the case of NF02 and TF03, the vertical gravity gradient was measured explicitly. For the other stations we employed the standard gradient of –3.086 µGal/cm. After data reduction, the drift of the LCR G-761 was analyzed graphically and found to be almost linear. Therefore we applied a linear drift model in the network adjustment.

Absolute gravity values

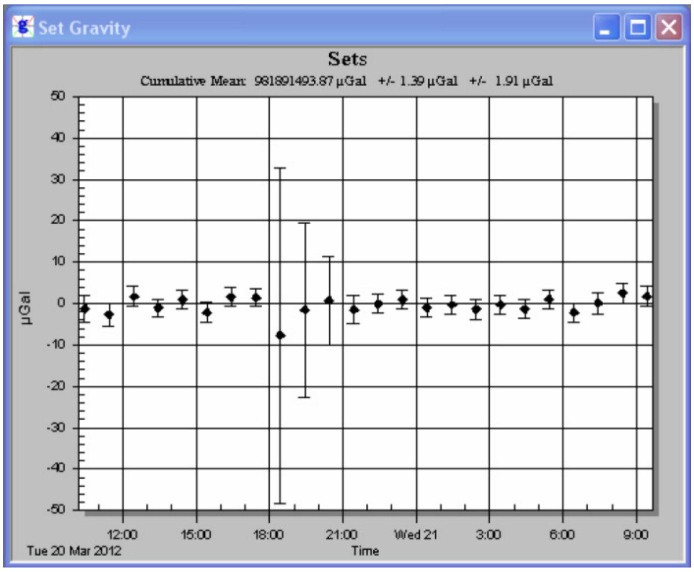
The results of the FG5 observations are shown in Table 1. The gravity values refer to the instrument reference height 1.2 m above floor level. For TF03 a weighted average of the three independent sessions was computed where the standard deviations of each session were used to generate weights. When referring the absolute values to the ground marker, the high accuracy of the absolute observation is degraded by the less precise vertical gradient. The estimated accuracies at the ground markers are 8.5 µGal at NF02 and 2.6 µGal at TF03.

The floor values for TF03 and NF01 are listed in Table 2.

Figure 5 shows the time series of 24 individual set values (average over 50 single drops) measured with FG5-226 at NF01. The three observations after 6 pm on 20 March 2012 show a significantly larger standard deviation than all other set values. This can be related to seismic disturbances caused by a large earthquake in the Oaxaca region of Mexico. In the software, the values were not considered for computing the final gravity value at NF01, since a reject threshold of $3 \cdot \sigma$ was exceeded.

Table 1: Results of absolute gravimetry

Site	φ	λ	H [m]	g [µGal]	σ [µGal]	number of sets	∂g / ∂H [µGal/cm]
NF01	59°39'59"	10°45'23"	66 ± 7	**981 891 493.87**	**1.39**	24	-3.03
TF03	59°39'56"	10°46'40"	95	981 884 409.27	2.28	24	-2.99
				981 884 410.11	1.53	14	
				981 884 408.80	2.19	18	
				981 884 409.59	**1.10**	(weighted average)	



Cumulative Mean: 981891493.87 µGal +/- 1.39 µGal +/- 1.91 µGal

Figure 5: Time series of individual set values during the 24 hours session at NF01 starting on Tuesday, March 20, 2012 around 10 am UT time.

Gravity network adjustment

The relative observations were adjusted by the method of least squares. The fundamental observation equation for relative gravity observations is given by

$$z_i = g_i - N_0 - D \cdot t - \Delta s \cdot z_i, \qquad (1)$$

where z_i are the raw (but scaled and tide corrected) relative gravity readings at station i, g_i are the corresponding unknown absolute gravity values, N_0 is the unknown instrument level (bias between relative readings and the absolute level), D is an unknown linear drift coefficient and Δs is a possible residual scaling factor. With 16 observations at the 6 stations and 3 additional unknown instrument parameters (N_0, D and Δs) the redundancy of the system is 7. However there is a rank defect of the design and the normal equation matrices and the system cannot be inverted directly. This singularity arises because relative observations cannot provide absolute values and absolute scaling of the network. The rank defect in this case is 2. Similar datum defects typically arise in leveling networks if absolute height values are to be derived from leveled height differences. The datum defect can be avoided for example by introducing additional conditional equations. Thereby the original normal equation matrix is padded by the corresponding conditional matrix. The conditional equations connect several or all of the unknown parameters. Alternatively one can fix the absolute values at selected absolute stations (the datum points) in a tight or loose fashion. Tight fixing reduces the parameter space by (in our case at least) two elements, namely by the gravity values of two datum points. The design matrix of the linear model is thus reduced for two columns. For illustration one might separate the original parameter vector $\boldsymbol{x}$ into the two components $\boldsymbol{x}_D$ and $\boldsymbol{x}_0$ where the first corresponds to the absolute values at the datum points. Then instead of the original linear model (formally separated into two components)

$$\mathbf{l} = \mathbf{A}\mathbf{x} = \left[\mathbf{A}_D \;\vdots\; \mathbf{A}_0\right] \begin{bmatrix} \mathbf{x}_D \\ \cdots \\ \mathbf{x}_0 \end{bmatrix} \qquad (2)$$

only the reduced system

$$\mathbf{l} = \mathbf{A}_0 \mathbf{x}_0 \qquad (3)$$

is solved. The respective elements of the observation vector $\boldsymbol{l}$ need to be reduced for the absolute value at the datum station, i.e. $l_i = z_i - x_{D,i} = z_i - g_i$. This solves the datum defect and the normal matrix of the reduced system is no longer singular.

The disadvantage of this type of tight fixing is that by decoupling the datum points from the adjustment model also their stochastic information is decoupled. The error estimates of all stations thus depends on the choice of the datum stations (datum choice) and existing error correlations between datum points and all other stations disappear. In addition, fixing more stations than the minimum number necessary for solving the datum defect leads to network deformations. In case of our gravity network the datum deficiency is 1 if we do not solve for the scaling factor Δs and 2 otherwise. Therefore one or two absolute stations need to be fixed to solve the datum defect depending on whether or not we estimate the scaling factor. Alternatively absolute stations can be fixed in a loose fashion. Thereby the absolute values are introduced as additional observations along with their stochastic information. Instead of reducing the parameter space (as in the case of tight fixing) the space of observations is extended. We separate formally the parameter vector into two groups of elements, i.e. $\boldsymbol{x}_{\mathrm{D}}$ for the gravity values at the loosely fixed datum points and $\boldsymbol{x}_0$ for the gravity values of the remaining stations. Let $\boldsymbol{A}_{\mathrm{D}}$ and $\boldsymbol{A}_0$ be the corresponding parts of the design matrix, then the original linear model

$$\mathbf{l} = \mathbf{A}\mathbf{x} = \left[\mathbf{A}_D \;\vdots\; \mathbf{A}_0\right] \begin{bmatrix} \mathbf{x}_D \\ \cdots \\ \mathbf{x}_0 \end{bmatrix} \qquad (4)$$

is extended to

$$\begin{bmatrix} \mathbf{l} \\ \cdots \\ \mathbf{l}_D \end{bmatrix} = \begin{bmatrix} \mathbf{A}_D & \mathbf{A}_0 \\ \cdots & \cdots \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_D \\ \cdots \\ \mathbf{x}_0 \end{bmatrix}. \qquad (5)$$

Thereby $\boldsymbol{l}_D = \boldsymbol{x}_D$. This way stochastic information on the quality of the datum information can be introduced. This is especially important in cases where the relative observations are more precise than the a priori absolute values at the datum stations. By tuning the stochastic information of the datum information one can change the relative weighting between the datum information and the inner geometry determined by the relative observations. This depends on the assumed quality of both. In our case the absolute gravity observables were assigned their estimated standard deviations and the relative gravity observables an assumed standard deviation of 15 µGal. We estimated the unknown gravity values at the network stations in two independent runs. In the first run we assumed the scale error of LCR G-761 being known. In the second run we explicitly estimated a residual scaling error. In the first case the rank defect is 1 and we are free to select either TF03 or NF02 or both as datum points. In the second case the datum defect is 2 and we need to introduce both points as datum points. The structure of the corresponding design matrix is shown in Figure 6. Each dot corresponds to a non-zero element. There are 6 unknown gravity values and three instrument parameters and thus the design matrix comprises nine columns. The last three columns reflect the three instrumental parameters N_0, D and Δs. They are part of all relative observation equations. The 16 relative observations are reflected in rows 1 to 16 of the design matrix. The last two rows correspond to the a priori absolute gravity values introduced at the two datum points TF03 and NF01. The structure of the design matrix reveals that instrumental parameters are not estimated for those two *pseudo observations*. With 18 observations and 9 unknowns the redundancy of the system is 9 and due to the two datum points there is no rank defect.

Table 2 shows the results of the adjustment (including scale error estimation). The scale error applied to the gravity difference between TF03 and NF02 (roughly 7000 µGal) results in a gravity difference of about 14 µGal. The same result is achieved in the second run where we did not estimate a scale error and introduced only NF02 as datum point. Then the difference between the estimated gravity value at TF03 (only based on relative observations tied to NF02) and the observed FG5 value also amounts to about 14 µGal. This value is comparable to the measurement uncertainty of the LCR instrument. We conclude that the scale error is not significant.

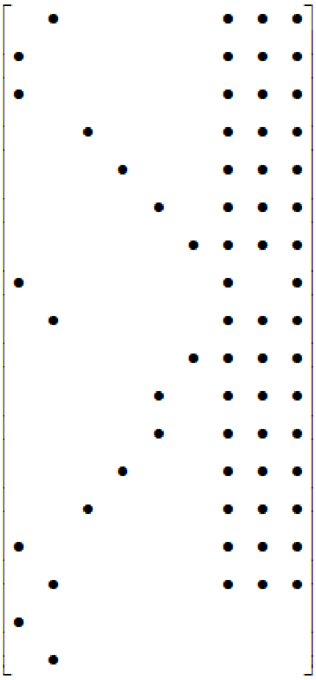


Figure 6: Structure of the design matrix when estimating a scale error

Table 2: Results of least squares adjustment, TF03 and NF02 (loosely) fixed, scale error estimated

	g-value (mGal)	$\hat{\sigma}_g$ (µGal)
TF03	981 884.7683	2.6
NF02	981 891.8574	8.5
SH01	981 884.3281	10.9
CI01	981 886.6672	10.0
UR01	981 887.5185	9.0
IT01	981 888.9019	10.8
N_0	981 885.4537	5.9
D	31.32 µGal/hour	3.8 µGal/hour
Δs	–0.001946 (≈ 1.9 ‰)	0.001787 (≈ 1.8 ‰)

Educational aspects and experiences

Observational work at the graduate level is designed towards several intentions and side effects. An observational program requires planning and practical choices. In the preparatory stage it is necessary to consider the challenges that will emerge during analysis. This may reveal special requirements of program design or critical aspects that limit the errors of the results. The handling of scientific instruments is not always straight forward and often reveals individual talent and skill. One reported side effect is the pleasure of mastering the practical work. Another is the personal discovery of how months (and years) of study in various topics is realized during various stages of the analysis. The interconnection of many topics to solve a specific problem is perceived in an inspirational way by a motivated student. Time limitations required small scale and specific problems to be solved in the current experiment reported here. It led to the core of a campus gravity network, which will be extended by future students. The principles apply to establishment of national and international gravity reference networks (carried out by mapping authorities in several countries).

Despite the small scale, the requirement for precise station descriptions is universal in geodesy. The hands-on experience with complex scientific equipment is challenging; this is a source of motivation and a possible recruiting mechanism for future scientists. The imperative role of observational uncertainty is experienced at first hand. The data are well suited for several applications of adjustment theory and demonstrates least squares adjustment as a versatile and powerful tool in geomatics.

The performance of the relative gravimeter in laboratory conditions may be a future topic. Analog and digital ways of recording the gravity value may be executed, and several instrument reading areas may be selected. An expanded test field (more realism and more data) allows different adjustment methods to be considered, e.g. conditional adjustment with constrains. Numerical experiments may be made with datum and rank deficiency, e.g. considering free datum parameters in gravity networks.

Conclusions

Modern research equipment has been made available to graduate students of geodesy. Several participating roles have led to project reports during the last five years. Here we have reported the results of establishing the core of a campus gravity network and applying it to test the scale error of a relative gravimeter. We experience that a gravimetric observational program combined with a comprehensive theory of the gravity field of the Earth helps make a cohesive curriculum in physical geodesy.

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