Bestemming av en høydereferanseflate ved bruk av en kvasigeoidemodell og GNSS/nivellementsdata

En undersøkelse fra Østlandet

Hossein Nahavandchi og Lars Skjeset

Vitenskapelig bedømt (refereed) artikkel

Hossein Nahavandchi og Lars Skjeset: Geoid-type surface determination using a gravimetric quasigeoid model and GNSS/leveling data- A case study in eastern Norway

KART OG PLAN, Vol. 73, pp. 211–231, POB 5003, NO-1432 Ås, ISSN 0047-3278

Norway is about to realize a new vertical datum NN2000, which is supposed to replace the present datum NN1954. Norwegian Mapping Authority (NMA) is also realizing a new geodetic reference frame ITRF09, with focus mainly on vertical accuracy. The present geodetic reference frame EUREF89 was realized as a horizontal frame. Norwegian Mapping Authority has earlier calculated geoid-type surfaces for converting ellipsoidal heights in EUREF89 to normal heights in NN1954. In this study, we have calculated geoid-type surfaces for the new vertical datum NN2000, by fitting the gravimetric quasigeoid model NKG2004 to Global Navigation Satellite System (GNSS)/leveling data in a test area in eastern Norway. The calculations were carried out by the geodetic gravity field modeling programs GRAVSOFT, where the fitting process to GNSS-levelling data are done by least squares collocation or weighted mean methods. The results from calculations show that the new vertical datum NN2000 yields improved accuracy compared to NN1954. Assessment of the geoid-type surface models computed from EUREF89/NN2000 and EUREF89/NN1954 shows Standard Deviations (SD) of 15 mm and 20 mm, respectively. Standard deviation of 20 mm for EUREF89/NN1954 is in accordance with empirical values of latest model calculated by NMA. Analogous fitting process to ITRF09/NN2000 gives standard deviation of 23 mm, indicating that ITRF09 does not yield improved accuracy. However, ITRF09 coordinates used in the calculations where preliminary, so final coordinates are expected to give more accurate geoid-type surfaces. Fitting to GNSS/leveling data is tested by varying number of GNSS/leveling stations, indicating that increasing number of stations does not automatically yield better accuracy for the estimated geoid-type surface models in this study. The accuracy of the model is relatively stabilized for 30 GNSS/leveling stations in test area in eastern Norway.

Keywords: Geoid-type surface, vertical datum, gravimetric quasigeoid, GNSS stations, GRAVSOFT

Hossein Nahavandchi: Division of Geomatics, Norwegian University of Science and Technology, NO-7491, Trondheim. E-Mail: hossein.nahavandchi@ntnu.no

Lars Skjeset: Division of Geomatics, Norwegian University of Science and Technology, NO-7491, Trondheim. E-Mail: skjeset@stud.ntnu.no

Introduction

The introduction of satellite technology in geodesy has modernized and increased the efficiency of the geodetic computations, significantly. It soon became common to use GNSS instead of traditional triangulation to determine the horizontal coordinates, but it took longer to replace the expensive and time consuming leveling by simple GNSS measurements for accurate determination of heights. The obtained height from GNSS is not referred to the quasigeoid but the ellipsoid. The relation among quasigeoid (ζ) derived from gravimetric models, ellipsoidal height (h) derived from GNSS and normal height H^N measured by leveling and referred to vertical datum is:

$$h - H^N - \zeta = e \qquad (1)$$

Acknowledging plumb line curvature and neglecting all data errors, the values of e should be zero, but rarely are zero due to various errors such as long wavelength quasigeoid errors, systematic errors in leveling networks, datum inconsistency between quasigeoid and the national vertical datum, and post-glacial rebound (e.g. Ekman 1989). As such, there are always discrepancies between a gravimetric quasigeoid model and the heights from GNSS/leveling. Therefore, using a gravimetric quasigeoid model to transform GNSS ellipsoidal heights to normal heights using Equation (1) does not always yield results that are compatible with the national vertical datum. (see e.g. Featherstone 1998, Nahavandchi and Soltanpour 2006). To improve the conversion of the GNSS heights to orthometric/normal heights (the heights desired by the users), the gravimetric quasigeoid model should be fitted to the GNSS/leveling data. The new combined surface (which is no longer an equipotential surface, thus no longer a classical gravimetric quasigeoid surface) can then be used to convert the GNSS ellipsoidal heights to normal heights. This new combined surface is called geoid-type surface in this study. The Norwegian Mapping Authority calls this surface height reference model. In Norway, the gravimetric quasigeoid models and related normal height system (see Section Normal height system 1954) are in use and Equation (1) is used in this study. The accuracy of the normal height determination using GNSS data is dependent on the accuracy of GNSS measurements and geoid-type surface model.

Numerous studies have been implemented to combining a gravimetric geoid/quasigeoid and GNSS/leveling data. For example Sideris et al. (1992), Jiang and Doquenne (1996), Kostakis and Sideris (1999), Featherstone (2000), Marti et al. (2002), Iliffe et al. (2003), Nahavandchi and Soltanpour (2006), Doquenne et al. (2005), Fotopoulos (2005), Featherstone and Sproule (2006), Soltanpour et al. (2006), Darbeheshti and Featherstone (2010), Klees and Prutkin (2010) and Šprlák (2010).

In Norway, the European Terrestrial Reference System ETRS89 realized in 1994/95 and was called EUREF89. This was realized as a horizontal reference frame, focusing on the horizontal coordinates. Height components were also, as the result of this realization, became available, without being so concerned about accuracy at that time. The demand for accurate height data, however, increased more than expected.

The investigations made by the NMA show that the accuracy of the ellipsoidal heights in EUREF89 does not meet the requirements for determining the coordinates by 1 cm accuracy in a national reference frame. This was the reason that NMA started working to establish a new framework for realizing the International Terrestrial Reference Frame (ITRF) in Norway, where height component was in focus. The basis for the new framework is the permanent geodetic stations.

With precise GNSS positioning, we first calculate ellipsoidal height differences, before using a geoid-type reference model to convert ellipsoidal height differences to height differences in national vertical datum. Geoid-type surface is generated using the common points that have both known ellipsoidal heights from GNSS measurements and normal/orthometric heights from leveling networks.

In this study, we calculate geoid-type surfaces based on ellipsoidal height in EUREF89 and ITRF09 and normal heights in NN1954 and the new vertical datum NN2000 which is about to be introduced in Norway. It will be examined whether the new reference frames ITRF09 and NN2000 provide better accuracy than the present EUREF89 and NN1954 frames. We also investigate the number of GNSS/leveling stations needed for the calculation of geoid-type surface in the test area.

Basic definitions

Geoid-type surface

To convert the GNSS ellipsoidal height to the normal height in the Norwegian national height system NN1954 (and later NN2000), one needs the converter model (geoid-type surface). NMA has determined several converter surfaces, named Height Reference Models (here we call it geoid-type surface). These surfaces are named HREFxx which convert ellipsoidal heights in EUREF89 to normal heights in NN1954. “xx” stands for the year model is released, and a letter that designate the models within each year chronologically. The last model, HREF2008a (in the time of this study) is valid for Norway's mainland with an accuracy better than 2 cm over large parts of the country (Norwegian Mapping Authority 2008).

The quasigeoid model describes the difference between a given reference ellipsoid and the quasigeoid, while a geoid-type model describes usually the difference between a given reference ellipsoid and (national) height datum. The geoid-type surface, thus, is a quasigeoid model which is adapted to (national) height systems using GNSS/leveling data. Quasigeoid can be calculated from the gravity data using Molodensky's well-known formula (Molodensky et al. 1962; Heiskanen and Moritz 1967, Eq. (8–10)). Geoid-type surface is calculated from the quasigeoid model and GNSS/leveling data.

NMA fits NKG96 quasigeoid model to the GNSS/levelling data to calculate different geoid-type surface models (Norwegian Mapping Authority 2008). The presentation of the models has occurred in parallel with the development of national network in the form of an iterative process where the model has been continuously updated as new areas of the national network have been completed. So far (at the time of this study), there have been published over 30 different geoid-type surfaces, which HREF2008a are the last one. The fitting process used 499 GNSS/leveling stations. 1814 GNSS/leveling stations are used to assess the estimated model. Figure 1 shows the HREF2008a model with the spatial distribution of GNSS/leveling stations used in the fitting process. The accuracy of the model depends partly on the distance and the distribution of the GNSS/leveling stations, and the accuracy of the NKG96 quasigeoid model. Control calculations with GNSS/leveling data that had not been used in the fitting procedure show a standard deviation of less than 2 cm. In areas where the distance to fitting stations is larger, the accuracy is slightly lower. In order to calculate normal heights from HREF2008a model, it is assumed that the GNSS ellipsoidal heights are given in EUREF89.

In this study, we determine geoid-type surfaces for the new vertical datum NN2000, by fitting the gravimetric quasigeoid model NKG2004 to GNSS/leveling data in a test area in eastern Norway. The Nordic gravimetric model NKG2004 was chosen since it was judged to be the best gravimetric model available for Norway in the study period. It has been computed by the remove-compute-restore method using a Wong and Gore type of Stokes' kernel and the residual terrain model reduction. The applied Global Geopotential Model (GGM) is a combination of the GRACE model GGM02S and EGM 96 (Forsberg et al. 2004).

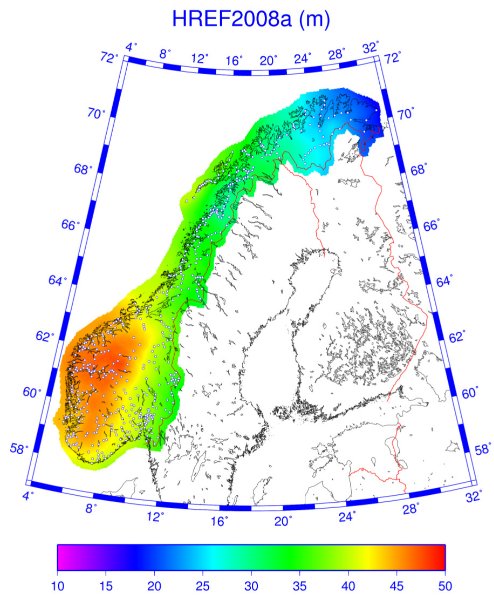

Figure 1. The HREF2008a model and the GNSS / leveling stations used in the calculation of this model.

International Terrestrial Reference System in Norway

A preliminary realization of the International Terrestrial Reference System (ITRS) in Norway is implemented through the Norwegian permanent geodetic stations, and 15 International GNSS Service (IGS) stations. Daily estimates from October 2008 to September 2009, with the exception of a period in winter where the antennas were covered with snow, form the basis for implementation. The reference epoch is set to 31 July 2009 (2009.578) and the reference frame is called ITRF09. In the summer and autumn 2009, it was carried out two measurement campaigns to estimate coordinates of 137 benchmarks in a test area in eastern Norway. Each point was measured twice with at least 48 hours of observation each time. It was emphasized to do the observations in two different period of time. Different receivers and antennas were used and the comparison between the first and second round of the observations revealed that the antennas must be calibrated. This work is still ongoing at the time of this study (Tangen, personal communication 2010). The coordinate values used in this study are only based on the first campaign and the results from antenna calibration were not available to be taken into account.

European Terrestrial Reference System in Norway

European Terrestrial Reference System (ETRS) is based on the definition of ITRS, with the same location of the origin and coordinate axes. The realization of the system differs, however, slightly from the realization of the ITRS. The first realization of ETRS called ETRF89 (Boucher and Altamimi 1992). This was based on the International Terrestrial Reference Frame (ITRF89) coordinates of ITRS stations in Europe. But since ETRF89 is fixed to the Eurasian tectonic plate which moves at 2.5 cm/year, thus ETRF89 moves away from ITRF89 at the same speed. For each new release of the ITRF, one can calculate a new ETRF from the coordinates of the ITRS stations in Europe, but it was decided to keep the first realization of ETRS, which was ETRF89, instead of frequently updating the frame.

ETRS89 was realized in Norway and named EUREF89 through GPS campaigns in the late summer and autumn 1994. 7 SATellite based Reference System (SATREF) stations in Norway were measured, in addition to the international campaign DOSE-NORD94 and the Norwegian campaign EUREF-NOR94 (Kristiansen and Harsson1996). Moreover, data from IGS stations in Ny-Ålesund and Tromsø and data from a GPS receiver that was placed at the Agricultural University in Ås in spring 1995 were used (See Figure 2).

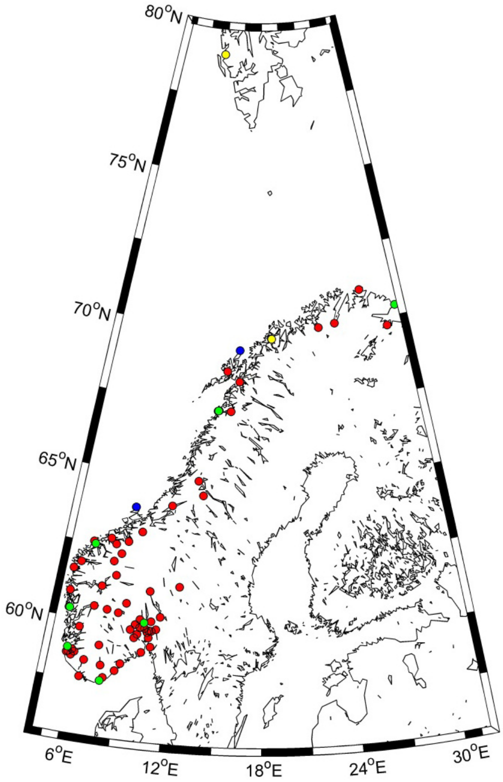

Figure 2. IGS (yellow, 2 stations), SATREF (green, 7 stations), EUREF-NOR94 (red, 63 stations) and DOSE-NORD94 (blue, 2 stations) stations used to realize EUREF89

DOSE (Dynamic Of the Solid Earth) was a project led by NASA to determine the horizontal and vertical movements of the last ice age. Two Norwegian stations (Andøya and Mausundvær) were included in the project. The EUREF-NOR89 was observed in 63 stations associated with the national network.

The calculations of the coordinates that realize the EUREF89 were done first with respect to ITRF93 at epoch t_c (the epoch when the GPS measurements were carried out). Then the data were transferred to ETRS89 at epoch t_c and finally to the epoch 1989.0. During the calculations, the coordinates of 15 stations in Europe (including 7 VLBI and 7 SLR stations) were held fixed. The uncertainty for each of the measuring stations was calculated by looking at how individual daily solutions distributed around the station's mean. The mean uncertainty for all stations was 2.2 mm in the northern direction, 3.1 mm in the eastern direction and 6.7 mm vertically. Detailed description of the calculations can be found in Kristiansen and Harsson (1996).

When EUREF89 was realized in 1994, it was mainly because of horizontal coordinates. The height component was just considered as a value with no concern on accuracy. However, the demands for heights increased gradually with time and the geodetic society started to use ellipsoidal heights in EUREF89 with a geoid-type surface to obtain normal heights in height system NN1954. The question that gradually arose was that whether it was necessary to have a better reference frame for the ellipsoidal heights. That is why the NMA is now working on a new reference frame on ITRS that focuses on providing good ellipsoidal heights.

Normal Height System 1954

NN1954 is the official Norwegian vertical datum. Efforts to establish a national height reference system in Norway started in the second half of the 1800s. In 1876 it was set up a tide gauge in Oslo to determine the average water levels over years and in 1890 data from this tide gauge was used to create a fundamental point at Norwegian Geographical Survey (NGO) in Oslo. At the same time, one started leveling along the main communication arteries in 1877, but the work became more effective only after the new modern equipment was provided in 1916. Yet the leveling network was not completed in southern Norway (south of latitude 66.3°) until 1953. At that time there were also set up new tide gauges along the coast. These showed annual land uplift along the coast, but the size varied from place to place. In Oslo, the annual uplift of approximately 3 mm was observed, so it was decided to move the fundamental point to Tregde near Mandal in southern Norway.

The height of the fundamental point is based on the adjustment of the mean water level calculations at the tide gauge stations in Oslo, Nevlunghavn, Tregde, Stavanger, Bergen, Kjølsdal and Heimsjø in 1956. In the adjustment, between 18 and 53 years of measurements of tide gauge stations were used and Tregde was chosen as the fundamental point because of its best characteristics in relation to the isostatic land uplift (See also Norwegian Mapping Authority 2002). The data from leveling were adjusted with fundamental point in Tregde as fixed, and this resulted in the national height system fixed Zero 1954 (NN1954). The realization of NN1954 is described in details in Trovaag and Jelstrup (1956).

The long period of almost 40 years to realize NN1954 led to a big problem: no reliable model for the land uplift was available, so the adjustment did not take into account this problem. Subsequent studies have shown that the land uplift in the period between the time of leveling and adjustment is of the order 15–20 cm at the most exposed areas (see e.g. Lysaker et al. 2006). This has led to a strongly deformed system. In addition, one aimed to pursue the quasigeoid as a reference surface, but because of insufficient gravity data in leveling network, the theoretical normal gravity from Clairauts formula was used. Comparison of NN1954 with ideally estimated normal height system and orthometric height system, both with reference epoch of 1954, showed that NN1954 is neither a normal system nor orthometric system. If we correct NN1954 for land uplift that took place from leveling time to adjustment time, the corrected height system, however, agrees well with normal heights (Lysaker et al. 2006) (see also Figure 3).

Normal Height System 2000

Large deformation in NN1954 and requests for more accurate heights by new technologies, e.g. areal laser scanning, led to demands for establishing a new vertical datum in Norway. Also, a vertical system that was more consistent with national datums in other Nordic countries was desired. In particular, Sweden and Finland have the same problems as Norway with respect to land uplift, and then a new height system that deals with this problem for the Nordic countries was requested.

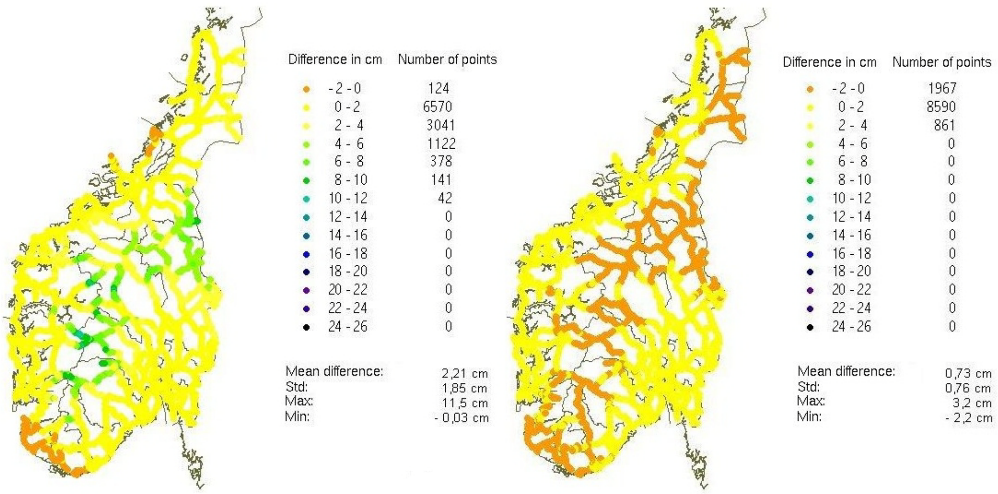

Figure 3. The difference between ideally estimated orthometric height (left) and normal heights (right) and NN1954 corrected for land uplift (Lysaker et al. 2006)

Political changes and the desire for a united Europe also led efforts to create a common European vertical datum. The first part of the work was UELN95/98, a leveling network that covers large parts of Europe and refers to the fundamental point Normaal Amsterdam Peil (NAP). The European Vertical Reference System (EVRS) was first realized as EVRF2000 using normal heights in UELN95/98 network. See Augath and Ihde (2002) for more details. After realization of EVRF2000, more data were produced by several participating countries and EVRF2007 was realized (Sacher et al. 2008). Because the Nordic leveling blocks were only linked to the rest of UELN95/98 via one network between Sweden and Denmark (two after the Øresund Bridge opened), then EVRF2000 could not be adopted directly. Moreover, the treatment of land uplift was not uniform in all countries where there was an actual problem. Therefore, it was decided that the Nordic countries were to implement a regional adjustment and use an appropriate land uplift model. To associate the Nordic block better to the rest of Europe, a new leveling network from Finland to Estonia were established (Mäkinen et al. 2005). This new network was called the Baltic Levelling Ring (BLR). Leveling data used in adjustment, called BLR2000, were EVRF2000 data from Estonia, Latvia, Lithuania, Poland, Germany and the Netherlands, and data from the last leveling campaign in Denmark, Sweden and Finland and all leveling networks in Norway (see Figure 4). The data were first transferred to a common reference epoch 2000.0 using land uplift model NKG2005LU (Ågren and Svensson 2007). NAP was held fixed, and NN2000 has been realized through the normal heights of 19,000 control stations across Norway that is part of leveling network. The calculations are based on 26000 km leveling network collected from 1916 to 2008 (Lysaker, personal communication 2009).

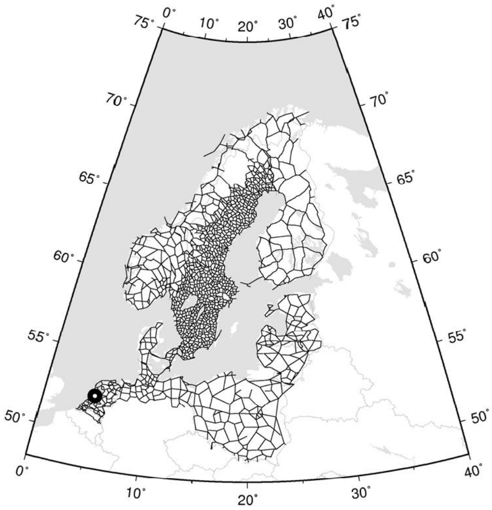

Figure 4. Leveling data used in the adjustment BLR2000 (Vestøl 2009)

To get insight into the differences between the new NN2000 and NN1954 height systems, 65 leveling benchmarks in eastern Norway are used. Figure 5 shows the differences between these two systems. It can be seen that the areas furthest east (Trysil and Engerdal) have the largest differences. This is because the area experiences large land uplift. The differences between NN2000 and NN1954 over Norway are from approximately –10 cm at the coast in southern and northern Norway to 30 cm in Trysil in the eastern Norway (Vestøl 2009). The introduction of the new height system NN2000 has just started. The plan is to recalculate the leveled benchmarks and the benchmarks that are not leveled (e.g. the benchmarks that are calculated by trigonometric methods).

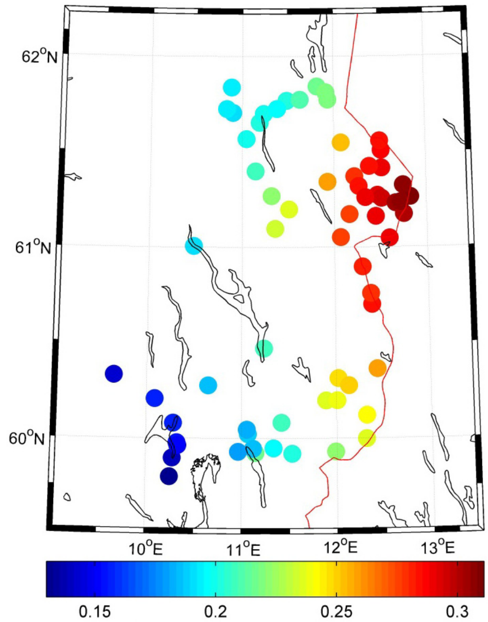

Figure 5. The differences between NN2000 and NN1954 at 65 benchmarks in eastern Norway in meters.

Numerical investigations

Data and geoid-type surface calculation method

The study area is located in eastern Norway limited to $59° < \phi < 62.5°$ and $8.5° < \lambda < 14°$. Numerical investigations are divided to a preliminary calculation and a final geoid-type surface model determination. The data for the investigation were made available by the Norwegian Mapping Authority, which are presented below:

- Gravimetric quasigeoid model of NKG2004 given as grid values in the area $53° < \phi < 73°$ and $1° < \lambda < 33°$ with resolution $\Delta\phi = 0.02°$ and $\Delta\lambda = 0.04°$.
- NN1954 normal heights at 146 stations in the test area (see Figure 6).
- NN2000 normal heights at 73 stations in the test area (see Figure 6). All stations with height values in NN2000 have height values in NN1954 too.
- Ellipsoidal heights in EUREF89 at 146 stations in the test area (the same stations that have normal heights in NN1954).
- Preliminary estimated ellipsoidal heights in the new reference frame ITRF09 (see Section International Terrestrial Reference System in Norway) at 137 stations in the test area. Stations are measured by the Ashtech UZ-12 or Netra Trimble receivers with ASH701945E_M, ASH700936E or TRM29659.00 antennas. The observation time in each point is at least 48 hours (Tangen, personal communication 2010).

Normal heights in NN1954 were used to investigate whether the new heights in NN2000 provide better accuracy compared to NN1954. The normal heights were not corrected for land uplift in preliminary computations. They were corrected for in final geoid-type surface model determination. Ellipsoidal heights in EUREF89 and ITRF09 were used for calculation of geoid-type surfaces. All 73 stations with estimated normal heights in NN2000 also have nomal heights in NN1954, and ellipsoidal height in EUREF89, but 7 of them have not the ellipsoidal height in ITRF09. In addition, a station has been moved and therefore gave large discrepancies when comparing height anomaly calculated by ITRF09 and NN2000 ($\zeta_{ITRF09/NN2000}$) and the corresponding height anomaly from NKG2004 ($\zeta_{NKG2004}$). This station was therefore excluded from the computations. Thus, there were 65 stations that were used in determination of geoid-type surfaces in NN2000 and ITRF09. Figure 6 shows the GNSS-measured stations in the test area that also have known normal heights in NN2000 and NN1954. The distribution of the stations in the north, east and south is relatively uniform, but compared to NN1954 stations, the NN2000 stations are not enough in the west and centeral part of the study area.

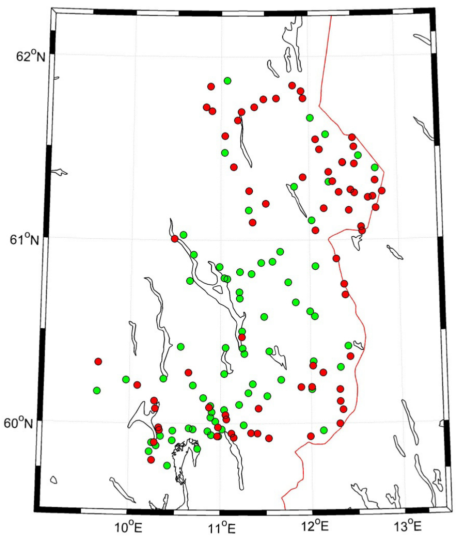

Figure 6. Measured GNSS stations with normal heights in NN1954 (green and red) and in NN2000 (red).

In calculation of geoid-type surface, we have used the GRAVSOFT package (see e.g. Forsberg and Tscherning 2008). GRAVSOFT is a collection of several FORTRAN programs for various computations within physical geodesy. GRAVSOFT contains several programs that are useful for this study, i.e. calculating the geoid-type surface. Two key programs to fit a quasigeoid to GNSS/leveling data are GEOIP and GEOGRID. GEOIP is a program for interpolating functional values at the computation points from a grid using bilinear or spline interpolation techniques and to use these interpolated values in arithmetic operations with other values at the same points. Spline interpolation is performed based on usually 8 x 8 points around the computation point. GEOGRID is a program which creates a regular grid from randomly distributed points. This is done either by Least Squares Collocations (LSC) or weighted mean techniques. Internally, the program uses a quadrant-based nearest neighbor search, so that only the *n* nearest points (maximum 25) in each quadrant is used to determine the value of the computation point. This is to increase the calculation speed. Since the LSC can only be performed on stationary data, therefore one has to remove the trend in data. There are different models available in GRAVSOFT to remove the trend from data: mean value model, linear model, second order polynomial, and third and fourth order polynomials.

The quasigeoid model used in the calculation of the geoid-type surface in this study is NKG2004. This model provides height anomaly values (ζ) at the computation points. The heights related to ζ values are normal heights (see Equation (1)). NKG2004 is a quasigeoid model of the Nordic and Baltic area determined by Forsberg et al. (2004).

We used following steps for geoid-type surface determination (see also Solheim 2000):

- The quasigeoid model NKG2004 gives us height anomaly values of ζ_n at computation points, which are GNSS/leveling stations. We then find the difference $\Delta\zeta = \zeta_{GNSS/leveling} - \zeta_n$. GEOIP program is used.
- We make a grid of $\Delta\zeta$ values using GEOGRID.

- We compute a geoid-type surface with adding the gridded values of $\Delta\zeta$ to the original gridded quasigeoid model of NKG2004 using GEOIP.
- We assess the accuracy of the new geoid-type surface model ($\zeta_{\text{geoid-type surface}}$) by calculating $\Delta\zeta_{\text{Control}} = \zeta_{\text{GNSS/leveling}} - \zeta_{\text{geoid-type surface}}$ using GEOIP.

This final assessment is carried out at the stations that are not used in the calculation of the geoid-type surface model. This normally gives less correlation. However, it should be mentioned that there are other techniques for control/assessment of the fitting process. Cross-validation technique is one of them (see e.g. Soltanpour et al. 2006).

There are different methods that can be used to create the grids from not-gridded datasets. The most common method is the LSC, but the weighted mean and second-generation wavelets transformation can also be used (see e.g. Soltanpour et al., 2006). Whatever method used, it is important to note that the addition of grid values in step 3 to the original quasigeoid model may provide systematic errors, especially in the areas that there are not enough GNSS/leveling stations. Therefore it is important to only use geoid-type surface models in the areas with dense and enough GNSS/leveling fitting stations, if accurate results are requested.

Results

The fitting of NKG2004 model to the GNSS/leveling data was done with several datasets and different methods. Initially, a simple fitting was made to see how the new normal heights in NN2000 works in comparison with NN1954, and also to determine which methods and parameters to be used in GRAVSOFT in the calculation of final geoid-type surface model. The datasets that were used are given in Table 1. The design of the test area, datasets and GNSS/leveling measurement campaign were carried out by the Norwegian Mapping Authority. This design was based on the area of interest which was in the eastern Norway and the GNSS/leveling data observed at the time of study.

Table 1. Datasets used in the preliminary computations.

Dataset	Normal height	Ellipsoidal height	Number of stations
1	NN1954	EUREF89	146
2	NN2000	EUREF89	73
3	NN2000	EUREF89	65
4	NN2000	ITRF09	65

Dataset 3 was included to make a test with exactly the same stations as in the dataset 4. In these preliminary calculations, the datasets were divided randomly into two, and half of the GNSS/leveling stations were used in the fitting process, while the other half was used to assess the computed model. As mentioned above, the fitting and assessment of the results were preliminary and the final calculations would be made later on. Therefore, the random division of the datasets into two would not affect the final results. However, it would give an indication how different parameters/methods would affect the results. For datasets 3 and 4 the division was the same, so that the same GNSS/leveling stations were used for fitting and control for both datasets.

The calculations were made using GRAVSOFT as described above. In GEOIP, we used spline intrepoltion technique using 8 × 8 points window around the computation point. In GEOGRID, we used different methods for removal of trend and creating grids from the random values. First, the data were gridded using least squares collocation method, using each of the 5 methods for the removal of trend (see Section Data and geoid-type surface calculation method). There were 25 points in each quadrant used for calculation of trend parameters. Correlation length and the Root Mean Square (RMS) noise needed in the LSC method (see e.g. Moritz 1980) was set to 50 km and 0.02 m, respectively. The second order Markov covariance model is used in the computations (see e.g. Moritz 1980 and Soltanpour 2007). Choice of the correlation length is based on values recommended in the manual for GRAVSOFT which are 50–100 km (Forsberg and Tscherning 2008). Soltanpour (2007) found that the correlation length for the similar computations were 54 km, 77 km and 122 km for third order polynomial, five parameter trigonometric model and linear model, respectively, using second order Markov covariance model. This agrees well with the recommended values in GRAVSOFT. A study in Australia, that also used the GRAVSOFT found an optimal correlation length of 7.5 km (Featherstone 2000). This study, however, had smaller distance between the GNSS/leveling stations. Manual for the GRAVSOFT has no recommended value for the RMS noise, but an example used the value of 0.02 m with correlation length 50 km. Featherstone (2000) found that the optimal RMS noise was 0.012 m.

Weighted mean interpolation technique was also used in the computations. We used third order polynomial to remove the trend in this technique, as it was the best method to remove the trend from data in LSC method. The power of interpoltation in weighted mean method was set to 2, which is the recommended value in GRAVSOFT. For more details see Forsberg and Tscherning (2008).

The grid results are limited to area 59° < ϕ < 62.5° and 8.5° < λ < 14° with resolution $\Delta\phi$ = 0.02° and $\Delta\lambda$ = 0.04°. Data with less than 10 km distance to the computation point was taken into account in calculation, but since all the points lied within the grid cells this had no practical significance. As described above, half of GNSS/leveling stations were used in fitting process and the remaining for subsequent assessment of the fitted model. Figure 7 shows the stations that were in each of the two groups, respectively, for dataset 1 (NN1954/EUREF89) and dataset 4 (NN2000/ITRF09).

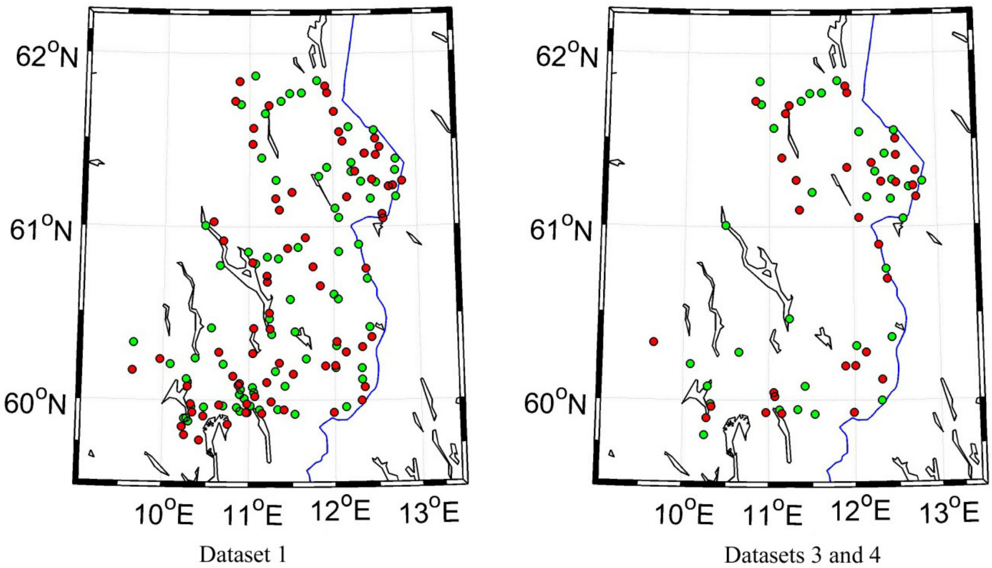

Figure 7. The GNSS / leveling stations used in the fitting process (green) and in the assessment of the estimated model (red) in datasets 1, and 3 and 4, respectively.

Tables 2–5 show the results of the preliminary computations. These computations include: i) direct comparison of height anomalies derived from original datasets ($\zeta_{GNSS/leveling}$) and the NKG2004 estimated height anomaly (ζ_n), ii) geoid-type surface determinations using LSC method with different trend models, iii) assessment of the estimated models using both LSC and weighted mean techniques. LSC gives an optimal estimate of the parameters when they have suitable statistical properties (nearly zero mean and small variance) and not contain any trend, therefore we have made a trend removal process prior to the LSC. Since the datasets are not corrected for land uplift in this stage, it is expected that the original datasets have relatively large values in terms of standard deviation. Original dataset 1 (which uses NN1954) has the highest mean and standard deviations, 191 mm and 58 mm, respectively, compared with other datasets (which use NN2000). The differences are significant, even if we know that the number of GNSS/leveling stations is doubled in datasets 1. Even here, we see that the new vertical datum NN2000 improves the accuracy better than NN1954 compared to NKG2004. One sees, however, that standard deviation decreases where trend is removed, especially with models of higher order. The datasets 2, 3 and 4 have standard deviations less than 30 mm before fitting to GNSS/leveling data, so the removal of the trend does not have the same impact as in dataset 1.

For all datasets, the removal of the trend using third order polynomial gives least standard deviation. This model will therefore be used further in the computations. The two rows at the bottom of each table for the assessment of the fitted model show that the LSC and the weighted mean technique gives almost the same results in terms of standard deviation. In Tables 3 and 4, one of the control points has a standard deviation around 4σ. The numbers in parentheses in these two tables show the values when this control point is omitted.

The results suggest that NN2000 provide better accuracy compared to NN1954, with an improvement of the standard deviation around 5 mm for datasets 2 and 3 in relation to dataset 1 (following the removal of the control points with standard deviation around 4σ). It should be noted that we have used twice as many GNSS/leveling stations in dataset 1 than in the other datasets, which theoretically should provide better accuracy. Table 2 shows a standard deviation of 20 mm for geoid-type surface model based on dataset 1, which agrees well with the empirical accuracy of the official geoid-type surface model of HREF2008a (Norwegian Mapping Authority 2008). Lysaker et al. (2006) showed that the preliminary normal heights in NN2000 agreed better with gravimetric quasigeoid models than in NN1954 in a direct comparison of height anomalies. The result of this study (ibid) suggested that even a geoid-type surface model based on NN2000 provides better accuracy than a model based on NN1954, although the improvement was not significant.

The ellipsoidal heights in ITRF09 do not, however, give better consistency with NKG2004 than the heights in EUREF89. Standard deviation of 23 mm (for ITRF09) against 16/17 mm (for EUREF89) is resulted (see Tables 3 and 4). The datasets 3 (EUREF89/NN2000) and 4 (ITRF09/NN2000) consist of same GNSS/leveling stations, so the geographical distribution of fitting and control points are the same. Normal heights are also the same. Thus, it is the ellipsoidal heights which result in better consistency with NKG2004 for dataset 3 than dataset 4. The coordinates in ITRF09 are only preliminary values and new values with better antenna calibration will probably give better accuracy.

Table 2. The statistics of the geoid-type surface computations using datasets 1 in mm.

Dataset 1 (NN1954/EUREF89, 146 GNSS/leveling stations)		**Mean**	**SD**	**Min**	**Max**
Original data $\zeta_{GNSS/leveling} - \zeta_n$ at 73 stations		191	58	90	297
Geoid-type surface model. Trend removed with:	Mean removal	0	58	–101	106
	Linear model	0	26	–58	64
	Second order polynomial	0	21	–65	47
	Third order polynomial	0	19	–57	50
	4-parameter model	0	25	–66	55
Model control at 73 stations	LSC and 3.order polynomial	–1	21	–60	44
	Weighted mean and 3.order polynomial	–1	20	–65	44

Table 3. The statistics of the geoid-type surface computations using datasets 2 in mm.

Dataset 2 (NN2000/EUREF89, 73 GNSS/leveling stations)		**Mean**	**SD**	**Min**	**Max**
Original data $\zeta_{GNSS/leveling} - \zeta_n$ at 37 stations		–25	23	–72	17
Geoid-type surface model. Trend removed with:	Mean removal	0	23	–47	42
	Linear model	0	19	–34	47
	Second order polynomial	0	19	–33	51
	Third order polynomial	0	14	–32	37
	4-parameter model	0	19	–31	50
Model control at 36 stations	LSC and 3.order polynomial*	–1(2)	23(16)	–97(–49)	25
	Weighted mean and 3.order polynomial**	–1(1)	22(15)	–98(–45)	20

*The number in parentheses denotes the values after the control point with deviation –9.7 cm is removed.
** The number in parentheses denotes the values after the control point with deviation –9.8 cm is removed

Table 4. The statistics of the geoid-type surface computations using datasets 3 in mm.

Dataset 3 (NN2000/EUREF89, 65 GNSS/leveling stations)		**Mean**	**SD**	**Min**	**Max**
Original data $\zeta_{GNSS/leveling} - \zeta_n$ at 33 stations		–24	23	–72	17
Geoid-type surface model. Trend removed with:	Mean removal	0	23	–48	41
	Linear model	0	19	–35	44
	Second order polynomial	0	18	–34	48
	Third order polynomial	0	15	–36	40
	4-parameter model	0	19	–31	49
Model control at 32 stations	LSC and 3.order polynomial*	–6(–3)	23(17)	–92(–47)	25
	Weighted mean and 3.order polynomial**	–7(–4)	22(16)	–93(–44)	20

*The number in parentheses denotes the values after the control point with deviation –9.2 cm is removed.
** The number in parentheses denotes the values after the control point with deviation –9.3 cm is removed

Table 5. The statistics of the geoid-type surface computations using datasets 4 in mm.

Dataset 4 (NN2000/ITRF09, 65 GNSS/leveling stations)		**Mean**	**SD**	**Min**	**Max**
Original data $\zeta_{GNSS/leveling} - \zeta_n$ at 33 stations		77	29	18	128
Geoid-type surface model. Trend removed with:	Mean removal	0	29	–59	51
	Linear model	0	18	–38	35
	Second order polynomial	0	16	–35	32
	Third order polynomial	0	14	–33	20
	4-parameter model	0	18	–39	37
Model control at 32 stations	LSC and 3.order polynomial	1	23	–55	64
	Weighted mean and 3.order polynomial	–1	23	–54	59

Direct comparison of original data in Tables 4 and 5 with NKG2004 shows that the dataset 4 provides large standard deviations. This was expected as the data is not corrected for land uplift. The standard deviation of the original data is higher for dataset 4 than dataset 3, but after removal of the trend, the standard deviations are approximately the same for the two datasets. Assessment of the estimated model in Table 5 gives larger standard deviations than in the fitting process. It seems likely that the inaccuracy in ellipsoidal heights in ITRF09 at GNSS/leveling stations is removed as a trend. Figure 8 shows the difference $\zeta_{\mathrm{GNSS/leveling}} - \zeta_{\mathrm{NKG2004}}$ at 65 stations that are included in datasets 3 and 4. The colors are adjusted so that the differences between the minimum and maximum of the color scale are the same for both plots and mean of each dataset is located in the middle of the interval. One can see that differences vary significantly between points that are close to each other, suggesting irregularities/errors in the height values. The variations may be slightly larger for the ITRF09 than EUREF89, but the difference is small.

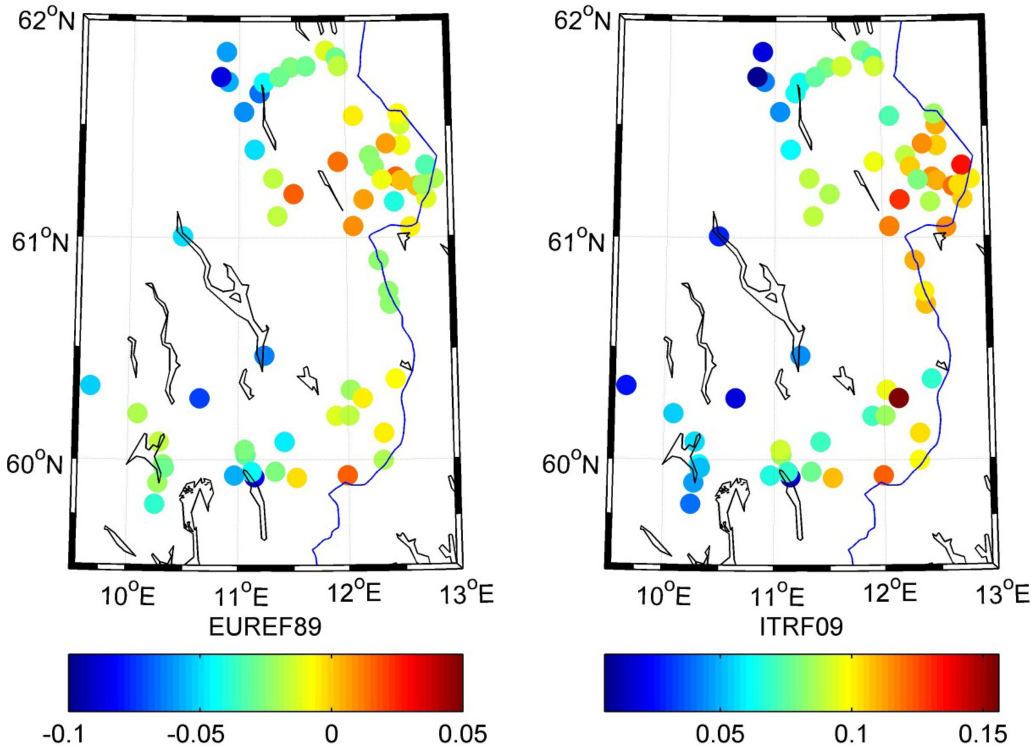

Figure 8. The differences $\zeta_{GNSS/leveling} - \zeta_{NKG2004}$ in meters for dataset 3 (left) and dataset 4 (right)

Figure 9 shows the differences $\zeta_{\mathrm{GNSS/leveling}} - \zeta_{\mathrm{NKG2004}}$ at the 65 GNSS/leveling stations in the datasets 3 and 4, as a function of length to the nearest GNSS/leveling fitting station and normal height, respectively. There is little difference between the two datasets, and the deviations do not appear to increase with increasing distance from the fitting points or increasing height.

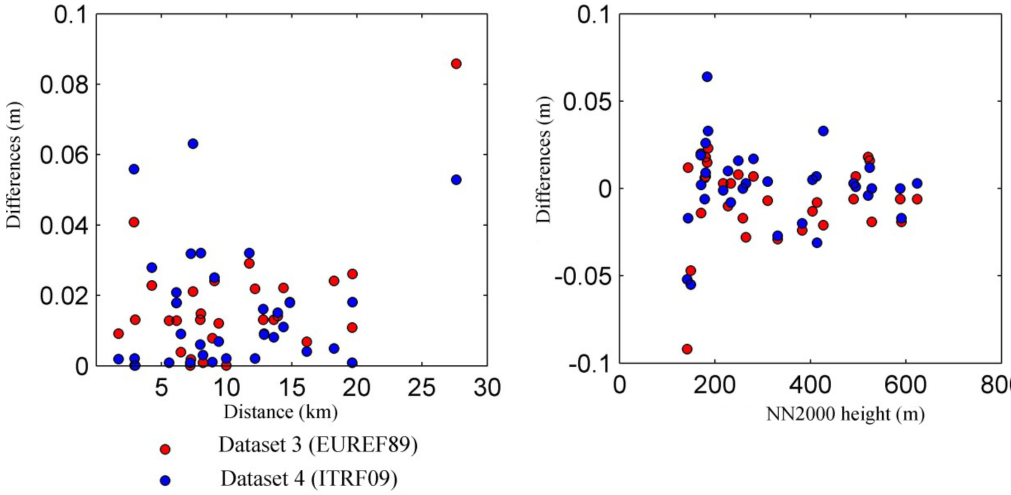

Figure 9. The differences $\zeta_{GNSS/leveling} - \zeta_{NKG2004}$ in meters for dataset 3 and 4 as a function of length to the nearest GNSS / leveling station (left) and normal height (right).

The results showed, as mentioned, little difference in accuracy between the least squares collocation and weighted mean methods. Since the LSC is the most common method used in the geoid-type surface determinations and it has been used for this purpose in Norway earlier, thus LSC is used in following for final calculations in the next step. However, it may be worth remembering that weighted mean technique is a good alternative to LSC method.

Finally, we investigated how the number of GNSS/leveling stations affects the fitting process and the accuracy of the resulted geoid-type surface model, and then we computed final geoid-type surface model. The height system used through final computations is NN2000. In these computations, we used third order polynomial for removing the trend effect and the least squares collocation method for geoid-type surface determination. The parameters in the GRAVSOFT were the same as previous computations. This investigation may give an indication on the number of GNSS/leveling stations needed in other test areas. The 65 stations in datasets 3 and 4 were used, and we also tested if the new ellipsoidal heights in ITRF09 provide better accuracy than in EUREF89. Test area with

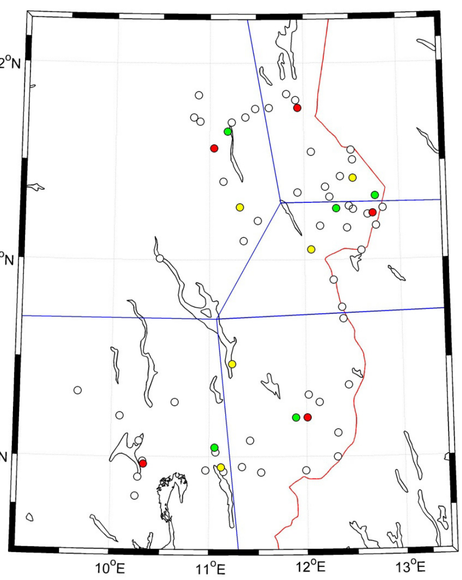

Figure 10. Five geographical areas with 13 GNSS / leveling stations in each area. White circles are GNSS / leveling stations used in the fitting process. There are three different sets of control stations: a) 5 red GNSS / leveling stations, b) 10 red and green stations, and c) 15 red, green and yellow stations.

GNSS/leveling stations was first divided into five geographical areas such as we have 13 GNSS/leveling stations in each area (see Figure 10). It should be noted that this design of geographical area would not affect the results, as all 65 GNSS/leveling stations will be used at the end in the computations. This division of the area is used to create an organized process of adding GNSS/leveling stations into the computation process. One may use other randomly-designed ways for testing the datasets. As also can be seen from Figure 10, there were areas with limited datasets where the distance to the nearest GNSS/leveling stations is slightly large.

One GNSS/leveling station in each area was selected as control point to assess the accuracy of the estimated geoid-type surafce. First, we used one stations from each of the five areas in the fitting process. Subsequently, 2, 3, 4,. . . , 10 GNSS/leveling stations from each area were used. This means that the fitting was made 10 times using 5, 10, 15,. . . , 50 GNSS/leveling stations in total. The remaining two benchmarks in each area was used either as fitting stations (so there were 55 or 60 fitting stations totally) or as control stations (so there were 5, 10 or 15 control stations totally). Thus, the control was always performed with GNSS/leveling stations that had not been involved in the fitting process. It was also possible to use GNSS/leveling stations involved in the fitting process for control (See e.g. Soltanpour et al. 2006). This investigation and the choice of the GNSS/leveling stations would give an indication of the accuracy of the estimated geoid-type surface model in this study. However, the findings of this investigation should be tested with other sets of GNSS/leveling data in other test areas.

The different height systems have different reference epochs and land uplift between those epochs should be taken into account in the final computations. Most of the land uplift maybe taken care of when one removes the trend, but it was more appropriate to correct the heights for land uplift in advance. The reference epoch of EUREF89 was first January 1989. It was 31 July 2009 for ITRF09, whereas NN2000 had the reference epoch of first January 2000. Therefore, we corrected the normal heights in NN2000 to the reference epoch of EUREF89 and ITRF09 for determination $\zeta_{EUREF89/NN2000}$ and $\zeta_{ITRF09/NN2000}$. The annual land uplift for each of the GNSS/leveling stations were taken from the NKG2005LU model which was based on leveling networks, tide gauges and permanent GNSS stations, combined with a geophysical model (Ågren and Svensson 2007). This model was a combination of Vestøls mathematical model (Vestøl 2006) and Lambecks geophysical models (Lambeck et al. 1998). Corrected normal heights in GNSS/leveling stations were calculated by:

$$\mathrm{NN2000}_{1989,0} = \mathrm{NN2000} - [(2000.0 - 1989.0)\frac{dH}{dt}] \qquad (2)$$

$$\mathrm{NN2000}_{2009.578} = \mathrm{NN2000} - [(2000.0 - 2009.578)\frac{dH}{dt}] \qquad (3)$$

where $\frac{dH}{dt}$ was the yearly land uplift for the particular station in question.

There are different ways to deal with the GNSS and levelling observations, e.g. in handling the permanent tide (see e.g. Ekman, 1989). This study uses tide-free or non-tidal model in the computations (a tide-free Earth with all direct and indirect effects of the Sun and Moon removed). It should be noted that formulas exist to convert the desired quantities (e.g. heights) between different tidal systems (see e.g. Ekman, 1989).

Tables 6–7 show the results of the geoid-type surface determination using 5 to 60 GNSS/leveling fitting stations. Datasets have been corrected for land uplift. As mentioned above, we decided to remove the trend by third order polynomial and perform the final geoid-type surface computations with the

LSC method. GEOGRID uses Cholesky decomposition method for the least squares solution, and when there were few fitting stations in relation to the number of trend parameters, singularity might occur. We have experienced singular/near-singular results where only 5 and 10 fitting GNSS/leveling stations were used in the least squares solution to remove the trend using third order polynomial (see row 3 in Tables 6 and 7). Not removing trend with 5 and 10 fitting stations resulted in large standard deviations. The standard deviation reached to 97 mm and 62 mm in Tables 6 and 7 with 10 GNSS/leveling control stations. Furthermore, we see that using 15 fitting stations and more, the estimated geoid-type surface model stabilizes relatively quickly. The standard deviations of the estimated geoid-type surface model, both for EUREF89 and ITRF09, were shown as a function of the number of GNSS/leveling stations in Figure 11. It was obvious from Figure 11 that using 5 and 10 GNSS/leveling fitting stations would result in large standard deviations, especially for EUREF89 in the test area. However, the standard deviations stabilized when the number of stations was increased. It is difficult to say, in general, how many GNSS/leveling stations are enough in the determination of a geoid-type surface model, but in the test area in this study, 30 fitting stations is a reasonable minimum. This can be seen from Figure 11. However, this result should be tested in other test areas with different GNSS/leveling stations.

Table 6. The statistics of the geoid-type surface model determinations in EUREF89 in mm, and the assessment of the estimated model.

EUREF89/NN2000	No. of Adjust. points	0	5	10	15	20	25	30	35	40	45	50	55	60
Original data	Mean		25	19	16	17	18	21	21	23	23	22	22	22
$\zeta_{GNSS/leveling} - \zeta_n$	SD		42	33	29	31	31	30	29	29	28	29	28	28
	Min.		−45	−45	−45	−45	−45	−45	−45	−45	−45	−46	−46	−46
	Max.		60	60	60	60	60	65	65	70	70	70	70	70
Geoid-type surface.	Mean				0	0	0	0	0	0	0	0	0	0
Trend removed by	SD				9	13	13	16	16	16	15	17	17	17
third order polynomial	Min.				−24	−26	−27	−32	−32	−33	−34	−48	−48	−47
	Max.				16	27	31	39	34	32	31	31	32	32
Control at 5 stations	Mean	17	8	−15	−6	−4	−4	−4	−5	−5	−5	−5	−4	−4
	SD	23	77	80	9	15	14	9	9	9	10	10	6	7
	Min.	−17	−78	−143	−21	−31	−26	−15	−18	−18	−20	−19	−14	−15
	Max.	34	84	78	2	6	11	6	3	4	4	3	2	2
Control at 10 stations	Mean	20	43	16	−6	2	3	−3	−4	−5	−5	−3	−3	
	SD	27	97	83	27	25	21	13	13	13	12	12	12	
	Min.	−21	−78	−143	−52	−31	−26	−25	−27	−28	−25	−25	−29	
	Max.	60	192	190	40	50	45	23	21	19	18	19	18	
Control at 15 stations	Mean	19	37	19	−8	−1	0	−4	−5	−6	−6	−4		
	SD	26	91	77	24	26	23	15	15	15	15	15		
	Min.	−21	−119	−143	−52	−55	−47	−26	−28	−28	−30	−30		
	Max.	60	192	190	40	50	45	23	21	19	18	21		

Table 7. The statistics of the geoid-type surface model determinations in ITRF09 in mm, and the assessment of the estimated model.

ITRF09/NN2000	No. of Adjust. points	0	5	10	15	20	25	30	35	40	45	50	55	60
Original data	Mean		40	38	34	34	36	37	37	40	40	41	41	42
$\zeta_{GNSS/leveling} - \zeta_n$	SD		42	35	31	35	32	30	29	28	27	29	28	28
	Min.		–29	–29	–29	–29	–29	–29	–29	–29	–29	–29	–29	–29
	Max.		69	71	71	86	86	86	86	86	86	113	113	113
Geoid-type surface.	Mean				0	0	0	0	0	0	0	0	0	0
Trend removed by	SD				12	13	14	13	13	13	13	17	18	18
third order polynomial	Min.				–20	–19	–30	–32	–30	–31	–31	–53	–55	–54
	Max.				18	26	24	23	22	21	20	50	53	54
Control at 5 stations	Mean	35	–5	–7	–2	0	0	–1	–2	–3	–3	–4	–4	–4
	SD	21	47	19	11	8	8	7	7	7	7	11	8	9
	Min.	6	–49	–38	–20	–12	–11	–11	–10	–13	–12	–21	–15	–14
	Max.	56	60	13	11	9	8	7	6	6	7	8	8	8
Control at 10 stations	Mean	40	28	–4	–6	0	1	0	0	–3	–3	–2	–2	
	SD	22	62	16	19	12	13	12	12	12	11	13	12	
	Min.	6	–49	–38	–48	–18	–13	–17	–19	–25	–24	–26	–26	
	Max.	74	129	13	11	16	22	18	19	14	10	11	14	
Control at 15 stations	Mean	42	26	–2	–5	0	1	0	0	–2	–1	–1		
	SD	22	55	20	22	18	17	17	16	17	17	19		
	Min.	6	–49	–41	–48	–29	–26	–28	–27	–29	–30	–31		
	Max.	85	129	29	39	32	26	25	26	25	30	32		

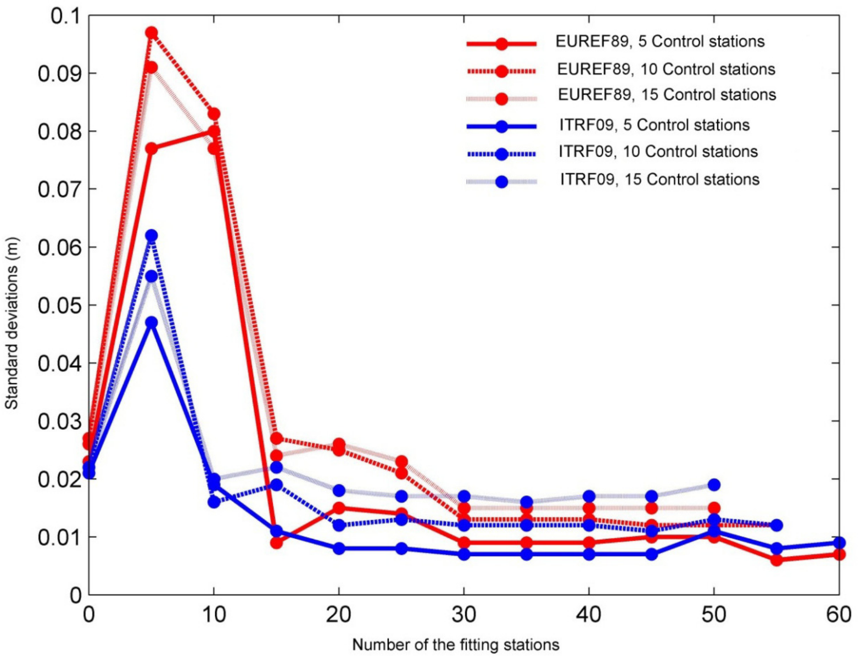

Figure 11. The standard deviation of the estimated geoid-type surface model at 5, 10 and 15 control stations versus the number of the fitting stations, for both EUREF89 and ITRF09

Figure 12 shows the final geoid-type surface model for EUREF89 and its deviation from NKG2004 quasigeoid model. One sees clearly that the deviation from the NKG2004 increases relatively rapidly in the areas without GNSS/leveling stations. It should be noted that the figures are only valid for the areas with GNSS/leveling stations. Numerical effects are dominant in the areas without GNSS/leveling stations. The deviation from NKG2004 displays smooth and long wavelength features which might be due to long wavelength errors in the global gravity model used in the geoid-computation process. Computational effects might also be the reason. Figure 13 shows the differences between geoid-type surface model using EUREF89 and two other geoid-type surface models. The left plot is the difference between model using EUREF89 and ITRF09 (in both models 30 fitting stations are used), while the right plot shows the difference between models using EUREF89 (with 30 fitting stations) and EUREF89 (with 50 fitting stations). Both plots show small values close to zero within the area with GNSS/leveling stations.

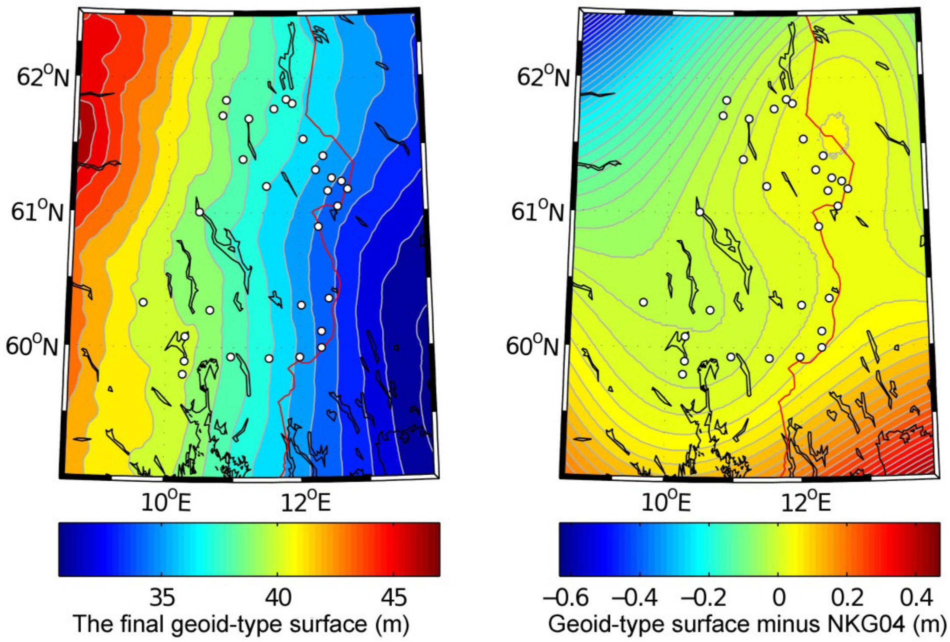

Figure 12. The final geoid-type surface model based on EUREF89 (left) and its deviation from the NKG2004 quasigeoid model in meter.

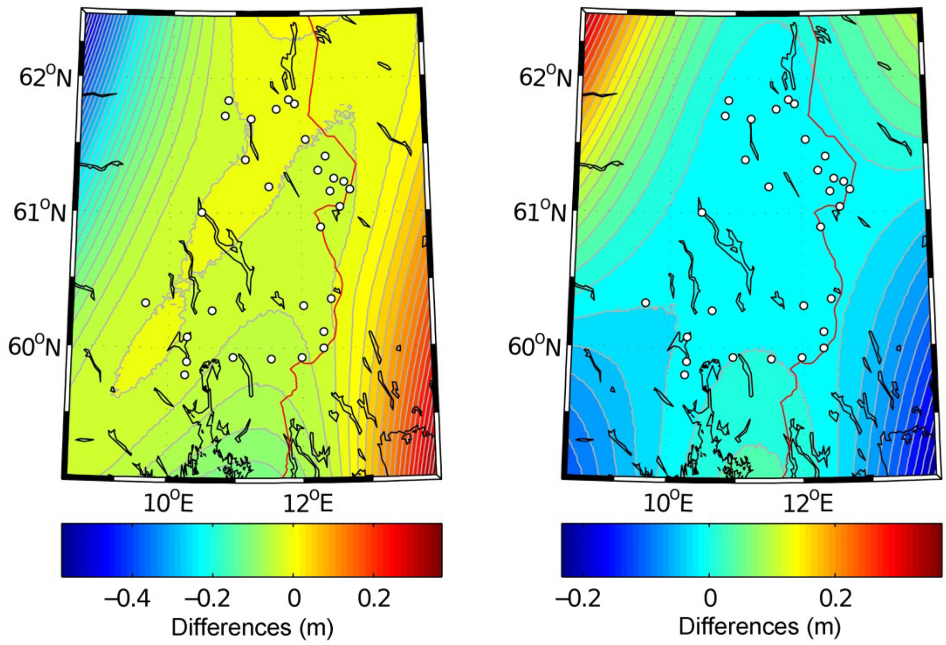

Figure 13. The differences between geoid-type surface models based on EUREF89 and ITRF09 (left), and based on EUREF89 with 30 and 50 GNSS / leveling fitting stations (right) in meter.

Conclusions

In modern height determination, ellipsoidal height difference is first calculated from a GNSS-vector, and a geoid-type surface model is utilized to convert this difference into a height difference in existing national vertical datum. This method replaces time-consuming and expensive leveling campaigns. The accuracy of geodetic datum where the GNSS-vector is referred to, national vertical datum, and geoid-type surface model, which describes the height difference between these datums, are the crucial factors in accurate height determination using GNSS/leveling data. A main goal in this study was to inquire whether the new reference frames NN2000 and ITRF09 yield better accuracy than the current reference frames of NN1954 and EUREF89 by geoid-type surface determination at several GNSS/levelling stations. In addition, we investigated how many GNSS/leveling stations were required to obtaining acceptable accuracy for the resulting geoid-type surface model.

Calculations of various geoid-type surface models, in this study, confirm that the new Norwegian vertical datum NN2000 improves the accuracy compared to gravimetric quasigeoid model NKG2004 than the old datum NN1954. Calculations were done in the test area in the eastern Norway and showed that the geometric height anomaly $\zeta_{\text{GNSS/leveling}}$ calculated from the ellipsoidal heights in EUREF89 and normal heights in NN2000 agrees much better to the gravimetric quasigeoid model NKG2004 than the corresponding geometric height anomaly calculated with normal heights of NN1954. The standard deviation improved from 58 mm (dataset 1) to 23 mm (datasets 2 and 3) by direct comparison $\zeta_{\text{GNSS/leveling}} - \zeta_{\text{NKG2004}}$. Dataset 1 consists of 146 GNSS/leveling stations with ellipsoidal heights in EUREF89 and normal heights in NN1954, while datasets 2 and 3 contain 73 and 65 stations, respectively, with heights in EUREF89 and NN2000. After fitting of NKG2004 quasigeoid model to GNSS/leveling data, the improvement, in terms of standard deviation, is from 21 mm to 16 mm, where large parts of the variations in the original data are eliminated in the fitting process.

Ellipsoidal heights in EUREF89, and in a newly realized reference frame ITRF09, were used in the computations. Coordinates in ITRF09 were only preliminary values based on one measurement in each benchmark. These showed no improvement compared with the old values in EUREF89. The calculations at the same benchmarks in both EUREF89 and ITRF09 gave $\zeta_{GNSS/leveling} - \zeta_{NKG2004}$ standard deviations of 23 mm and 29 mm, respectively. After fitting to the GNSS/leveling data, standard deviations were 17 mm and 23 mm in EUREF89 and ITRF09, respectively. Benchmarks with ellipsoidal heights in ITRF09 are now being measured again (at the time of the study), and new values based on both measurements will be available soon.

Furthermore, the study includes calculations with different numbers of GNSS/leveling fitting stations to examine how the resulted geoid-type surface model is affected by the number of points. The results of this study showed that the accuracy stabilized relatively quickly around 30 GNSS/leveling points. In order to determine how many GNSS/leveling stations to be used in other parts of the Norway, we would recommend doing similar calculations as in this study.

Acknowledgements. The authors are greatly indebted to the two reviewers of this paper for patiently reading the paper and for valuable comments. This work was done at the Norwegian Mapping Authority and the Norwegian University of Science and Technology. The authors wish to thank the Norwegian Mapping Authority and Dr. Dagny Lysaker that provided all the data for this research. The authors are indebted to Dr. Lysakr and the Norwegian Mapping Authority for guidance and helpful input. This study was supported by the Norwegian University of Science and Technology.

References

Augath, W., Ihde, J. (2002) *Definition and realization of vertical reference systems – The European solution EVRS/EVRF 2000*. Proceedings, FIG XXII International Congress, Washington, 19–26 April 2002.

Boucher, C., Altamimi, Z. (1992) The EUREF Terrestrial Reference System and its first realizations. Proceedings, Symposium of the IAG Subcommission for the European Reference Frame (EUREF), Bern 4–6 March 1992.

Darbeheshti, N., Featherstone, W.E. (2010) Tuning a gravimetric quasigeoid to GPS-levelling by nonstationary least-squares collocation. *Journal of Geodesy* 84: 419–431.

Duquenne, H., Everaerts, M., Lambot, P. (2005) Merging a gravimetric model of the geoid with GPS/levelling data: an example in Belgium. In: Jekeli, C., Bastos, L., Fernandes, J. (eds.) *Gravity, Geoid and Space Missions*. Springer, Berlin Heidelberg New York, pp. 131–136.

Ekman, M. (1989) Impacts of geodynamic phenomena on systems for height and gravity. *Bulletin Géodésique* 63: 281–296.

Featherstone, W.E. (1998) Do we need a gravimetric geoid or a model of the base of the Australian Height Datum to transform GPS heights? *The Australian Surveyor* 43: 273–280.

Featherstone, W.E. (2000) Refinement of a gravimetric geoid using GPS and levelling data. *Journal of Surveying Engineering* 126: 27–56.

Featherstone, W.E., Sproule, D.M. (2006) Fitting AUSGeoid98 to the Australian height datum using GPS data and least squares collocation: application of a cross validation technique. *Survey Review* 38: 573–582.

Forsberg, R., Tscherning, C.C. (2008) *An overview manual for the GRAVSOFT geodetic gravity field modeling programs*. Second edition. LAP Academic Publishing, Saarbrücken, Germany.

Forsberg, R., Strykowski, G., Solheim, D. (2004) *NKG-2004 Geoid of the Nordic and Baltic Area*. Proceedings, International Association of Geodesy Conference "Gravity, Geoid and Satellite Gravity Missions", Porto, Portugal, August 30–September 3, 2004.

Fotopoulos, G. (2005) Calibration of geoid error models via a combined adjustment of ellipsoidal, orthometric and gravimetric geoid height data. *Journal of Geodesy* 79: 111–123. DOI: 10.1007/s00190-005-0449-y.

Heiskanen, W., Moritz, H. (1967) *Physical geodesy.* Freeman, San Francisco.

Iliffe, J.C., Ziebart, M., Cross, P.A., Forsberg, R., Strykowski, G., Tscherning, C.C. (2003) OGSM02:

A new model for converting GPS-derived heights to local height datums in great Britain and Ireland. *Survey Review* 37: 276–293.

Jiang, Z., Duquenne, H. (1996) On the combined adjustment of a gravimetrically determined geoid and GPS levelling stations. *Journal of Geodesy* 70: 505–514. DOI: 10.1007/s001900050039.

Klees, R., Prutkin, I. (2010) The combination of GNSS-levelling data and gravimetric (quasi-) geoid heights in the presence of noise. *Journal of Geodesy* 84: 731–749.

Kotsakis, J., Sideris, M.G. (1999) On the adjustment of combined GPS/levelling/geoid networks. *Journal of Geodesy* 73: 412–421. DOI: 10.1007/s001900050261.

Kristiansen, O., Harsson, B.G. (1996) Nytt geodetisk grunnlag EUREF89 – GPS-kampanjen 1994 og analyseresultater, EUREF-NOR94. *Kart og Plan* 56(2): 104–115.

Lambeck, K., Smither, C., Ekman, M. (1998) Tests of glacial rebound models for Fennoscandinavia based on instrumented sea- and lake-level records. *Geophysical Journal International* 135: 375–387.

Lysaker, D.I., Omang, O.C.D., Pettersen, B.R., Solheim, D. (2006) Quasigeoid evaluation with improved levelled height data for Norway. *Journal of Geodesy* 81: 617–627.

Marti, U., Schlatter, E., Brockmann, E.,Weget, A. (2002) The way to a consistent national height system for Switzerland. In: Adam, J., Schwarz, K.P. (eds.) *Vistas for Geodesy in the New Millennium*. Springer, Berlin Heidelberg New York, pp 90–95.

Molodensky, M.S., Eremeev, V.F., Yurkina, M.I. (1962) *Methods for the Study of the External Gravitational Field and Figure of the Earth*. Washington DC: Office of Technical Services, Department of Commerce.

Moritz, H. (1980) *Advanced physical geodesy.* Wichmann, Karlsruhe.

Mäkinen, J., Lilje, M., Engsager, K., Eriksson, P.O., Jepsen, C., Olsson, P.A. (2005). *Regional adjustment of precise levellings around the Baltic*. Proceedings, Symposium of the IAG Sub Commission for Europe (EUREF), Wien 1–3 June 2005.

Nahavandchi, H., Soltanpour, A. (2006) Improved determination of heights using a conversion surface by combining gravimetric quasi/geoid and GPS-levelling height differences. *Studia Geophysica et Geodaetica* 50: 165–180.

Norwegian Mapping Authority (2002) *Norwegian Official Height Systems and Reference levels*. Standard document, Version 2.0.

Norwegian Mapping Authority (2008) *Norwegian Mapping Authority's Height Reference model, HREF.* Product information document.

Sacher, M., Ihde, J., Liebsch, G., Mäkinen, J. (2008) *EVRF07 as realization of the European Vertical Reference System*. Proceedings, Symposium of the IAG Sub Commission for Europe (EUREF), Brüssel, 18–21 June 2008.

Sideris, M.G., Mainville, A., Forsberg, R. (1992) Geoid testing using GPS and levelling (or GPS testing using levelling and the geoid?). *Australian Journal of Geodesy, Photogrammetry and Surveying* 57:62–67

Solheim, D. (2000) *New height reference surface for Norway.* Proceedings, Symposium of the IAG Sub Commission for Europe (EUREF), Tromsø 22–24 June 2000.

Soltanpour, A. (2007) *Geoid and geoid-type surface determination- Improved Determination of Geights from Global Positioning Systems*. Doctoral Theses at Norwegian University of Science and Technology 2007:24.

Soltanpour, A., Nahavandchi, H., Featherstone, W. (2006) The Use of Second-generation Wavelets to Combine a Gravimetric Quasigeoid Model with GPS-levelling Data. *Journal of Geodesy* 80: 82–93.

Šprlák, M. (2010) Fitting quasigeoid model using conventional and moving least squares methods. *Geodetický a kartografický obzor* 56: 246–255 (in slovak).

Trovaag, O., Jelstrup, G. (1956) *Precise leveling in south Norway.* Technical Report 5, Norwegian Geological Survey.

Vestøl, O. (2006) Determination of postglacial land uplift in Fennoscandia from levelling, tidegauges and continuous GPS stations using least squares collocation. *Journal of Geodesy* 80: 248–258.

Vestøl, O. (2009) *Innføring av nytt høydegrunnlag*. Local geomatics days, Lillehammer, 17–18 February 2009.

Ågren, J., Svensson, R. (2007) *Postglacial land uplift model and system definition for the new Swedish height system RH 2000*. Technical report LMV-Report 2007:4, Swedish Mapping Authority.